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Evaluation of bond graph based object oriented approach to determination of natural frequencies of packaging system elements

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Evaluation of Bond Graph Based Object Oriented Approach to Determination of Natural Frequencies of Packaging System Elements

by
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ABSTRACT

Packaged items passing through distribution system encounter vibrations. It needs to be examined what happens to an object, packaged or unpackaged, in the distribution environment when it is subject to outside vibrations, so that we may predict its behavior resulting from distribution vibrations. To prevent product loss from both static and dynamic forces encountered in distribution, corporations are turning in increasing numbers to services of package engineering.

This work evaluates the application of Bond Graphs Based Object-Oriented Approach to Determination of Natural Frequencies of Packaging System Elements. Using the Bond Graph, a model of the Package-product system under testing was developed to closely simulate the real testing model and conditions. The testing model equipment and packaging system were developed using the Bond Graph computer assisted modeling approach. Using simulation, vibrations characteristics of the packaging model were obtained, i.e., natural frequencies and other relevant properties of the system (model).

An analysis of the frequency characteristics was conducted by application of physical modeling using a suitable visual computer development platform. The results obtained by simulation, and those obtained on the vibration table (32,9 Hz vs. 26,777 Hz) are close to each other, they are very encouraging, and they open further possibilities.

This work has proved that the Bond Graphs based approach has its place in a computer aided total Packaging Distribution Design process. It should also enable one to shorten the entire design/testing process, to insure that those expensive parts (products) are not destroyed during the testing process, and that the finished package design meets the design objectives. This work has proved that satisfactory results could be obtained by computer simulation employing the Bond Graph methodology.

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1. Introduction

To prevent product loss due to the static and dynamic forces encountered in distribution, corporations are turning in increasing numbers to the package engineering. As new markets open for sophisticated consumer goods, such as the ubiquitous personal computer, corporate decision makers are becoming sensitive to the relationship of effective, protective packaging to product and company well-being. Although most obvious in the case of electronic products, protective package engineering is also being applied in many other product areas, from pharmaceuticals to fruit [1].

Product-package testing machinery and instrumentation, and the expertise necessary for efficient package development are increasingly in demand as package engineering keeps improving. As with any innovation, progress with more sophisticated concepts, methods, and procedures requires a corresponding increase in knowledge, understanding, and technical skills [1].

Bond graphs as model representation language are used for computer simulation and analyzes of dynamic engineering systems. They are well suited for modeling behavior of a complex multidisciplinary system in a unified way. Recent developments of this methodology based on object-oriented approach and symbolic computational algebra support have widened, even more, the capabilities of this methodology to deal with real engineering systems. Consequently, they can be used for simulation of testing product behavior (characteristics) when they are exposed to different kinds of stress, including shocks and vibrations.
This work evaluates the application of Bond Graphs Based Object-Oriented Approach to Determination of Natural Frequencies of Packaging System Elements.

1.0. Goals of This Work and How They Will be Achieved

Packaging system testing procedure will be reviewed. The packaging system model will be also analyzed and testing objectives will be determined. A model of testing equipment and packaging system will be developed using Bond Graph computer assisted modeling approach. The model of the system should be developed in such a way that vibration characteristics can be predicted with reasonably good accuracy. Using simulation, vibrations characteristics of the packaging system will be obtained, i.e., the natural frequencies and other relevant properties of the system (model). The results obtained by simulation will be compared to those obtained on the testing machine (the same model has already been tested on a testing machine at the R.I.T.). Finally, conclusions will be drawn regarding results achieved, and feasibility of using the Bond Graph modeling and simulation approach as an aid in a complete packaging system design process.

1.1. About Bond graphs

Bond graphs (BGs) were born in their present form on April 24, 1959. (Professor Henry M. Paynter gave a seminar on the subject, "Ports, Energy and Thermodynamic Systems" at Case Institute of Technology, Cleveland, Ohio, USA). His lectures to students were gathered later into the 1961 MIT press book "Analysis and Design of Engineering Systems."

His efforts[1] were motivated by his preoccupation with the logical philosophy underlying analogies in general. Such concerns were much earlier formalized by the mathematician, Eliakim Hastings Moore, in the following dictum:

"We lay down a fundamental principle of generalization by abstraction: *The existence of analogies between central features of various theories implies the existence of a general theory which underlies the particular theories and unifies them with respect to those central features...* "

It took nearly 20 more years before the BGs became widely known and employed. The individuals primarily responsible for this promotion were, most notably, Dean Karnopp, Ronald Rosenberg, and Jean Thoma.

While originally intended for modeling engineering systems, they have meanwhile found widespread application in many areas of physical system modeling[2]. BGs are a very appealing tool for modeling physical systems, because they represent the flow of power through a system. Since energy and mass are the only tradable goods in our physical universe, a BG model is more likely to reflect physical reality than a model derived by using any other modeling methodology.

Furthermore, the dynamics and control of complex mechanical systems and advanced mechatronic systems can be investigated more efficiently by using BGs. The mechatronics systems are usually complex and combine mixed technologies and thus mechatronic design requires a good modeling technique applicable across a wide range of physical domains[3]

1.2. Bond graphs and Packaging Technology

The functional representation of engineering systems (and of packaging systems, too) describes how the system works [2]. The design process should involve exploring design alternatives, simulation, and possible redesign in order to achieve stated goals. Among criteria for making a design decision, meeting all functional requirements is a vital one.

The explanation of how system works in its environment is based on a model. Modeling is usually done manually and *ad hoc*. It is a labor intensive and time consuming process difficult to validate and reuse.

Automated modeling approach (and tools), on the other side, reduce human involvement in modeling process and ensure that modeling principles are followed, and that models developed are more likely to be conceptually correct.

The BGs are recognized as important techniques in automated modeling, system decomposition and composition, simulation and, quantitative reasoning. They have been developed as a unified approach to the modeling of dynamic systems in engineering by using a standard set of entities to describe physical phenomena. The models based on BGs are more compact and more easily interpreted using representation methods such as block diagrams,

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[1] Henry M. Paynter: *The Gestation and Birth of Bond Graphs*; http://www.hankpaynter.com/Bondgraphs.html

[2] François E. Cellier: *Hierarchical Non-Linear Bond Graphs: A Unified Methodology for Modeling Complex Physical Systems*, Department of Electrical and Computer Engineering, The University of Arizona; http://www.ece.arizona.edu/~cellier/bondgraph.html

[3] Jozef Wojnarowski: *Bond graphs and mechatronic systems*, Silesian Technical University, Department Mechanics, Robots and Machines, Gliwice, Poland; http://www.ipme.ru/mirrors/GAMM/gamm98/num_abs/a632.html

signal flows or directed graphs. By using BGs it is possible to construct knowledge base containing models of engineering components based on several standard elements.

Packaging systems can be viewed as a collection of interconnected components. Each component on its own usually consists of simpler components and so on. To represent the model of a packaging system, a generic model of a component should be defined first. The BGs and Bond diagrams serve as model representation language. The basic item of a model is a component represented as an object which contains all information on its properties and behavior. The models are held in a database which supports common model management operations. The study of the system behavior can be conducted by simulation. Prior to simulation the model is processed. The integration of the system equations is provided by a suitable differential algebraic equation solver.

1.3. Distribution Packaging

The goal in distribution packaging (transport packaging) is to provide a correct design for packaging so that its contents arrive safely at destination (it does not include packaging for consumer goods, such as the primary packaging for food, beverages, pharmaceuticals, and cosmetics) without using too much or too little packaging materials.

The design of distribution package must have that goal in mind. Since distribution packaging must always be economical it should balance production protection, ease of handling and storage, shipping efficiency, manufacturing efficiency, ease of identification, customer needs, and environmental responsibility to achieve the lowest overall cost.

1.3.1. Hazards in Distribution

The product must be protected during distribution from damage or destruction and, likewise, if the product is hazardous, people and property must be protected from the product. For consumer goods, the retailer expects to receive an attractive, unblemished, salable package containing an undamaged product. The package-product system should be designed to withstand the rigors of the distribution environment. From the point of the product's creation, the product-packaging system must survive the hazards encountered until it reaches its final destination, at which time the packaging may be discarded or recycled. The hazards of distribution are many and varied, and it is usually difficult or impossible to predict exactly what a product-package system is going to encounter [1].

1.3.2. Distribution Environment

The first step in designing an effective package system is to determine the severity of the shipping environment [3]. Information on distribution environment provides the packaging professionals with the data on which to base decisions in these areas. Improvements in distribution equipment, operating procedures, and recording devices over the last several years have resulted in a need for new data to establish or reconfirm shock and vibration environmental profiles of today's distribution systems. However, caution should be expressed before heading out and measuring the environment. Defining environments is not as simple as shipping a few data recorders through a distribution channel. It is a complex, long, drawn out study requiring more resources than typically one company can invest [4].

Evaluation of the product's distribution method can determine which hazards the product is likely to come across, as well as the level of intensity of those hazards. Then the package system can be designed accordingly.

Package handling, transportation, and storage can lead to a variety of hazards within the shipping environment, including, but not limited to, vertical drops, horizontal impacts, vehicle vibration, temperature extremes, and compression loads. The method of distribution greatly influences the presence and severity of these hazards, so understanding the shipping environment is essential to designing a package that will effectively protect its product.

There are four different ways of determining the environment through which a product is shipped: observation, damage claims, literature search, and direct measurement.

These techniques can be used individually or in conjunction with each other [3].

1.4. Vibrations

As a package moves through distribution, we need to be concerned with the dynamic forces encountered due to manhandling (dropping, throwing and other abuses due to manual loading, unloading and movements of packages), warehouse handling equipment (stresses applied by mechanical handling equipment such as forklifts, conveyors, etc.), vehicle impacts (starting, stopping and other jolts due to the movement of trucks, railcars, ships, and aircraft), vehicle vibrations (the naturally occurring vibrations resulting from the motion of engines and moving contact of the vehicle with highway and rails) [1].

These four sets of conditions may result in impacts and vibrations to our product-package system. Package systems may be designed to minimize the damage caused in distribution, but sometimes the product itself must be redesigned in order to survive [1].

Vibration is associated with all transportation modes, although each mode has its own characteristic frequencies and amplitudes. Frequencies above 100 Hz are of little concern to most packagers, because in most packaging situations, the product will become isolated (that is, its vibration output will be less than the input received) at these higher frequencies. The most troublesome frequencies are below 30 Hz because they are most prevalent in vehicles, and it is difficult to isolate products from them [5].

Truck vibrations, for example, occur predominantly at the natural frequencies of the load on the suspension system, of the unsprung mass of the tires against the suspension system, and of the trailer and body structure. They are exacerbated by the condition and irregularities of the roadbed, the engine and drive train, tire and wheel imbalance, and the dynamics of the loading, or freight [5].

Vibration damage often occurs due to relative motion. Reducing or eliminating relative motion (wherever one part is free to move against another) lessens this type of vibration damage. Tight shipping case dimensions, particularly in the vertical axis, are preferred wherever this is compatible with top load compression of product and package [5].

Vibration resonance occurs whenever the forcing (input) frequency is the same as the natural frequency of the product and/or the package system. Resonance exists not only for the total assembly, but also for parts or subsections within the total structure. For protective packaging purposes, all resonance points should be located and quantified. This is done by subjecting the product to a range of frequencies and by observing the frequencies at which a resonance condition occurs. For packaging purposes, a typical resonance search might sweep the frequencies between 3 – 100 Hz at 0.5-1.0 octave per minute (refer to ASTM D 999). The search should be done in all axes [5].

Damage caused by resonance vibration can be difficult to resolve. The problem is complicated in that all cushioning materials are resilient and, while they are acting to attenuate the shock, they are also acting as a spring responding to vibration input. For many applications, it has been found that redesigning the product to eliminate critical resonance points is the most cost-effective method of decreasing damage. The measure of last resort is to design a vibration-isolation cushioning system [5].

Occasionally, entire loads go into a stack resonance condition, where each succeeding container goes into resonance with the previous container until the entire stack is bouncing, creating conditions likely to cause excessive damage [5].

Materials used to isolate from vibration are for the most part the same as those used to isolate from shock. An ideal vibration-isolation provides isolation in the 3-100 Hz range, since these are the predominant frequency ranges that cause damage during transport. However, cushioning materials, like all other springs, also have characteristic resonance points. A properly selected isolation material resonates at an input frequency that is less than half of the product's resonance frequency [5].

1.4.1. Distribution vibrations

Vibration is encountered in everyday life. Packaged items passing through distribution system encounter vibrations. It is those complex mechanical vibrations, and the accompanying changes in acceleration, which may lead to physical damage. It is necessary to examine what happens to an object, packaged or unpackaged, in the distribution environment when it is subject to outside vibrations, and we would like to predict behavior resulting from distribution vibrations [1].

Vibration is associated with all transport modes, although each mode has its characteristic frequencies and amplitudes. Vibration damage can take several forms. Scuffing and abrasion can occur wherever one part is free to move against another. Many particular products sifts or settle when vibrated and this leaves an open void at the top of the boxes and bottles, which the consumer invariably interprets as an under-fill.

The energy developed on the output side during a resonance condition can do many things: fatigue and finally fracture metal cans and pails; flex and crack delicate circuits on circuit boards; disintegrate or otherwise alter the texture of food products; separate and settle granular components in a food product or settle loose protective fill; aggravate scuffing and abrasion problems by several orders of magnitude; cause individual containers or components to bang into one another; disturb pallet patterns or dunnage (load-securing) systems; initiate stack resonance; unscrew bottle caps and threaded fasteners.

The greatest vibration input in a typical truck is directly over the rear wheels and tailgate. If damage is restricted to, or is most severe in that section of the vehicle, vibration inputs are almost certainly to blame. Vibration inputs are also usually the source if damage seems to occur only in the product layer next to the pallet or at the top layers [5].

1.4.2. The Natural Frequency

When a force is applied to a system setting it in motion and then withdrawn, the system will vibrate at a precise frequency. This frequency is defined as **natural frequency (f_n)**. **Spring-mass system** shown in Figure 1. will vibrate with the same frequency once it is set into free motion. It will oscillate according to the displacement equation:

$$x = A \sin (\omega t) \qquad (1)$$

where ω is a circular frequency.

The mass on a spring, shown in Figure 1., is an ideal model for a product in which we would like to predict the behavior resulting from distribution vibrations (product on cushions or structural components in products).

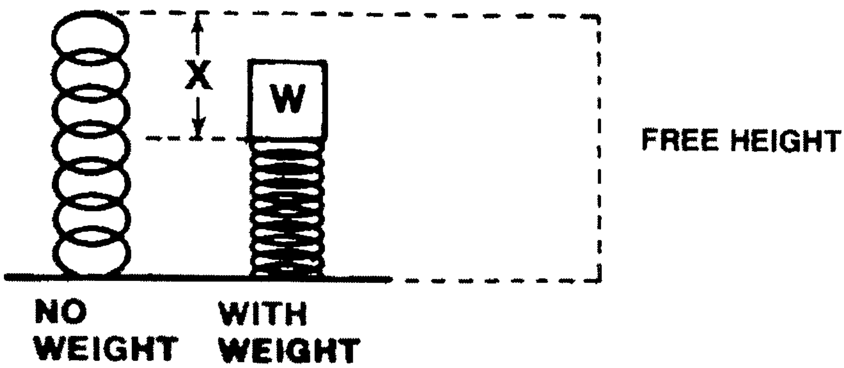

Figure 1. A Mass on a Linear Spring

1.4.3. Vibrations of Packaged Products

For packaging we are concerned with the maximum velocity and with the maximum acceleration of a product. If we have a product of mass m (kilograms or lb) supported by a piece of cushion material, we assume that it behaves as a linear spring. At rest the cushion is depressed (δ_{ST}) from the level of the unloaded condition. If we set the product-cushion system into motion by depressing the product downward from the resting position and then releasing it, the product (or mass) will vibrate indefinitely between the depressing distance and the initial position, if we assume that there is no friction or damping.
We may also approximate the motion of a trailer truck traveling on a highway by the sinusoidal motion of a linear spring-mass system (truck body and the load are the mass, and the suspension system is a linear spring; although these springs are really not linear, the useable parts of the springs when working together are very close to being linear). An external force sets this spring-mass system in motion when truck goes over a pothole, expansion joint, or some other road surface discontinuity.

From empirical investigation and mathematical analysis three general statements relating the input and output vibrations may be made [1]:

The output vibration will occur with the same frequency as the forcing vibration (not at the natural frequency of vibration). There may be other frequencies present in the vibration response to the forcing vibration, but the amplitudes and accelerations associated with these frequencies are much smaller than those of vibration at the forced frequency, f_f).

The output amplitude of vibration of the spring-mass system is directly related to the input amplitude of the forcing vibration by a calculable number, the magnification factor:
Output Amplitude = Input Amplitude x Magnification Factor

The maximum acceleration experienced by the spring-mass system is directly related to the maximum acceleration associated with the forcing vibration multiplied by the same magnification factor:
Maximum Output Acceleration = Maximum Input Acceleration x Magnification Factor

Figure 2 shows a product-package system. The product has a small internal component, which is known to have a natural frequency (this may be a small circuit board, transistor or similar mechanical/electrical part).
When the product is placed in the package a cushion deflection (δ_{ST}) is observed.

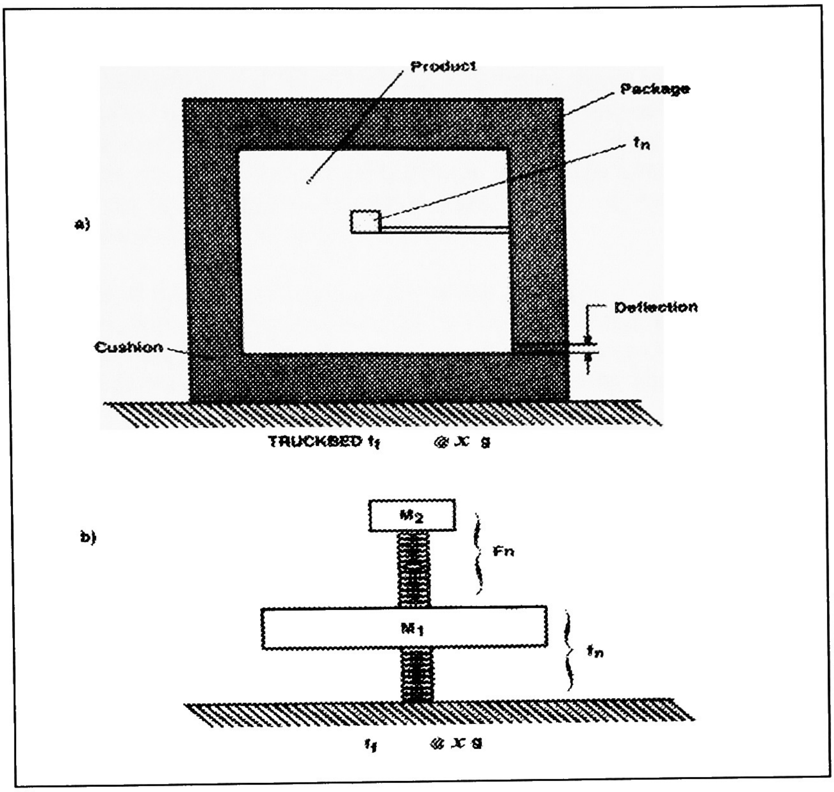

Figure 2. A Product-Package System

To simplify matters and help formulate the problem the model shown in Figure 2b may be used (the product is depicted as a mass (M_1) on a linear spring representing the cushion).

A product-package system reduced to its simplest form is shown in Figure 3.

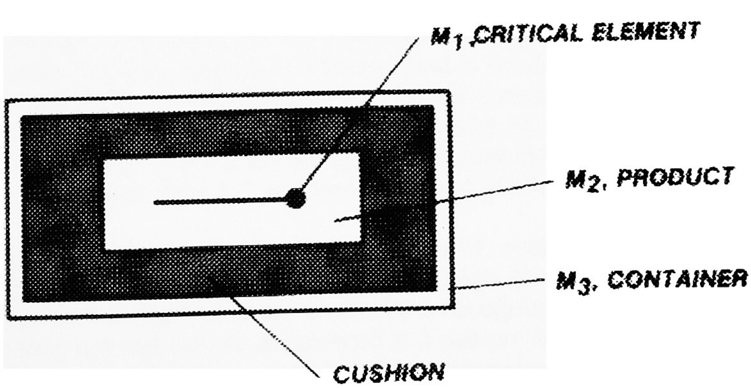

Figure 3. A Simple Product-Package System

The product-package system consists of four basic components: the outer container, the cushion, the product, and a critical element. The critical element is the most fragile component of the product (e.g., the filament of the radar tube). It is the part that is most easily damaged by a mechanical shock or by vibrations.

1.4.4. Forced Vibrations and Product Sensitivity

The vibration sensitivity of a product may be defined as those input vibration frequencies of sufficient amplitude occurring in the distribution environment, which can cause the resonance and failure of the product, or of a component within the product. Information on the vibration sensitivity may be used either to properly design a package system that will minimize the product's response to the input vibrations in distribution, or to redesign the product. The three-step method is used to determine vibration sensitivity:

- Identification of the natural frequencies of the product
  Identification of the natural frequencies which occur in the distribution environment
  Determination weather failure occurred when the product was vibrated at the sensitive frequencies at the amplitudes encountered in the distribution environment.

To develop a precise, low-cost package for product protection, some fundamental information about the product is needed. To protect the product from shock, a fragility level must be determined. Fragility is the maximum acceleration and velocity change the product can withstand before damage occurs. This information is charted to form a damage boundary curve. A product generally has different damage boundary curves in each orientation of the product. Ideally the fragility level is determined experimentally through a test procedure such as the American Society for Testing Materials (ASTM) D 3332 "Test Method for Mechanical-Shock Fragility of Products, Using Shock Machines." Due to product availability and the time needed to perform a fragility assessment, the fragility level is often derived through estimation.

Once the shock fragility is known for the product, cushioning material that provide the necessary protection can be selected. Historically, the use of cushion curves helps a designer identify the material; and thickness and loading range based on a pre-determined drop height and required acceleration level. Ranges of thickness, impact and drop heights are generally provided for a given material. By comparing cushioning curves of various materials (generated using the same test method), a cushioning material with the least amount of surface area and thickness can be chosen.

Ideally the effects of vibration on the product should also be investigated when designing a cushion system. Products typically contain a number of components, each having a unique response to the vibration input. By performing a vibration test on the product without packaging, the dominant response frequencies of the product elements can be identified.

Resonant frequencies of products can be identified by using the ASTM D 3580 "Standard Test Method of Vibration (Vertical Sinusoidal Motion) Test of Products" or the ASTM D 5112 "Standard Test Method for Vibration (Horizontal Linear Sinusoidal Motion) Test of Products." The objective in designing a package is to avoid overlapping product element resonance, as well as amplification range of the cushioning material (per loading and thickness) and significant environmental input frequencies for the intended mode of transportation.

Since the vibration test equipment is quite sophisticated, cushion designs are often created based solely on shock. Vibration evaluation of a package generally occurs once vibratory damage produced in the distribution environment is recognized. Limited data on vibration is another reason why it is not initially considered in a design. Most previously published data has become obsolete due to changes in manufacturing processes for many of today's cushioning materials.

Additional factors besides shock and vibration, such as economics, material availability, volume requirements, etc., also play a role in selecting a cushioning material. However, shock and vibration characteristics are the key elements [6].

1.4.5. New Trends in Vibration Testing

New trends in vibration testing are[4]:

- Using Environmental data obtained from field data recorder
- This data is then inputted into a computer controlled vibration machine

This type of technology brings the actual shipping environment the product experiences into the laboratory and enables us to recreate the damage potential the package would encounter. Large companies are doing vibration testing before shock or drop testing, i.e., they design for vibration protection first (since a product always experiences vibration and studies show that high drop heights are unlikely). In other words, vibration is certain, whereas shock or dropping is probable. When a compromise between shock and vibration is made, most companies will now chose vibration protection first.

From the data obtained from the field data recorder a dynamic vibration cushion curve can be developed. In the near future vibration cushion curves based on geometric shapes will be available to the packaging industry (in-package test method).
Packaging designers face many challenges in predicting the performance of cushioning materials when designing protective packaging. A prime objective is to choose the proper cushioning material that would protect the product from hazards of the distribution environment (shock and vibration). An additional goal is to achieve the most cost-effective package possible under given parameters. Historically, shock attenuation characteristics for cushioning materials have been presented as a material characteristic only (ignoring other variables in the package -- the container, friction, and air-flow dynamics) which may result in (or lead to) a package that fails to optimize cost and/or performance.

Shock attenuation is commonly presented as a cushioning curve, which is a plot of peak acceleration in G's vs. static loading, and is used to compare and qualify materials and package designs. The method in which cushioning data are derived determines how it can be used for a given purpose. While a material characteristic test is suitable for material comparisons and qualifications, it is not certain how applicable it is for package design.

A major supplier of protective packaging solutions has developed in-package test methods for evaluating shock and vibration characteristics of cushioning materials over the past two decades. To better predict package performance compared to traditional methods Package designers can use data based on these methods [6].

Though there is no industry-accepted test method for determining the vibration responses of cushioning materials for packaging, there have been many attempts to create such a method. Cushioning response characteristics to vibration are known as vibration transmissibility. Vibration transmissibility of a cushioning system is expressed as a non-dimensional ratio of its response amplitude to the excitation amplitude. The ratio may be one of force, displacement, velocity, or acceleration. The shape of a transmissibility curve is dependent on the degree of damping in the system.

The closest the industry has come to adopting a standard for vibration transmissibility -- though it was far from being accepted -- was in the late 1970s. The U.S. Air Force published an extensive amount of data on cushioning materials based upon a test method it developed. The test method and data were published in the MIL-HDBK-304B, "Military Standardization

[4] http://www.repco.com/Points/vibration.htm

Handbook, Package Cushioning Design." Problems associated with the military method, which tried to replicate a true single-degree-of-freedom spring-mass model, included the introduction of unwanted noise into the system and preventing the test block from leaving the cushion sample.

Adopting the principles discussed in Part I (PT&E, March '97, p. 26) [6] for the in-package test for shock, a major supplier of protective packaging solutions has developed a test method for generating vibration transmissibility data on cushioning materials. Using the same in-package set-up, the test package is placed on a vertical sinusoidal vibration machine. A frequency sweep from 5 to 150 Hz and back to 5 Hz is conducted at one octave per minute at a 0.5 G input (the rate and input is referenced to the ASTM D 999 "Standard Methods for Vibration Testing of Shipping Containers"). A vibration accelerometer is attached to the test block to measure the vibration response through the cushion system. The frequencies at which coupling, resonance, and attenuation take place can be identified for the thickness, loading, and material under evaluation.

It is possible there could be different frequencies for coupling, resonance, and attenuation between the sweep from 5 to 150 Hz and the sweep from 150 to 5 Hz. This occurs when the cushion under test breaks down during amplification. On the second sweep, the transmissibility curve is generally offset to the low frequency side of the initial curve, thus an average of the two frequencies is considered for coupling, resonance, and attenuation [6].

Computer Aided Engineering (CAE) methods are used for computer simulation and analyzes of dynamic engineering systems. They are well suited for modeling behavior of a complex multidisciplinary system in a unified way. Recent developments of this methodology based on object-oriented approach and symbolic computational algebra support have widened, even more, the capabilities of this methodology to deal with real engineering systems. Consequently, CAE methods can be used for simulation of testing product behavior (characteristics) when they are exposed to different kinds of stress, including vibrations and shocks.
Using simulation it should be possible to predict the behavior of a system subjected to vibrations. Results obtained by employing modeling and simulations should then be compared to the results obtained on the testing table (experiment).

Bond Graphs based approach in computer aided total Packaging Distribution Design process should speed up the complete design process by continuous evaluation by simulation after each design/change step, so that the finished package design could meet design objectives. That approach should also enable us to shorten complete design/testing process, and also spare those expensive parts (products) from destruction during the testing process (e.g., testing for resonant frequencies).

2. Vibration Testing

Natural frequencies of a product are usually identified using a Vibration Table capable of producing sinusoidal motion. The product is securely attached to the table, and the system is vibrated at frequencies between 3 and 100 Hz with an associated maximum acceleration of between 0.1 to 0.5 g's (zero to peak amplitude). A constant 0.5 g's is commonly employed throughout the frequency range sweep. The natural frequencies of the products are those which cause components to resonate, identifiable either visually or by the intense sounds of the vibration. The frequency sweep may be interrupted and the frequency kept constant to confirm the resonant frequency and the component affected. This procedure simply identifies the natural frequencies of a product in the range of 3 to 100 Hz. The natural frequencies may or may not be the breaking points of the product [1].

Table 1. Vibrations Encountered in Distribution[5]

Transport	Frequency (Hz)	Maximum G Level
Rail Car	Suspension frequency 2-7 Suspension, lateral ¾-2 Structural 50-70 Roll approx. 1	½ g ¾ g ¼ g -
Truck	Suspension 2-7, 4 Unsprung suspension 10-20 Structural 50-100	½ g ¼ g ¼ g
Trucks on Flat Cars	Vertical 2-4.6 Roll 0.7-3.1	1 g 10 g
Aircraft	Propeller 2-10 Jet 100-200	-
Ships	Sea and Engines 10, 100	

It is important to emphasize that if any of the natural frequencies of the product match up with frequencies occurring in the proposed distribution system, careful product testing is needed.

2.1. Vibration Test Methods

Standard test methods for vibration test of products are defined by the ASTM standards. The test methods cover the determination of resonance of unpacked products and components applied to the surface on which the product is mounted for test. Information obtained from the test methods may be used for product modification. This may become necessary if a product's response requires design of an impractical or excessively costly shipping container.

Referenced ASTM Documents for vibration test methods are:

D 3580	Test Method of Vibration (Vertical Sinusoidal Motion) Test of Products
D 4332	Practice for Conditioning Containers, Packages, or Packaging Components for Testing
D 5112	Test Method of Vibration (Horizontal Linear Sinusoidal Motion) Test of Products
E 122	Practice for Choice of Sample Size to Estimate a Measure of Quality for a Lot or Process
D 996	Terminology of Packaging and Distribution Environments

A vibration test machine consists of a flat horizontal test surface of sufficient strength and rigidity so that the applied vibrations are essentially uniform over the entire test surface. The test surface should be driven so as to move only horizontally/vertically along a single axis in sinusoidal motion (rotary motion is not acceptable). The frequency and amplitude of motion should be variable and under control to cover the specified range (test intensities should be sufficient to vibrate the product at acceleration and frequency levels that determine if product resonance occurs in the expected frequency range of the transportation environment). Experience has shown that individual transportation environments may contain frequencies ranging from 0.2 to above 100 Hz. Acceleration levels sufficient to excite resonance normally range from 0.1 to 0.5 g. [7] [8].

Specimen-Mounting Devices of sufficient strength and rigidity are required to attach the product securely to the test surface. The resonant frequency of mounting device should be above the intended test range for the product. The device(s) should support the product in a manner similar to the way in which it would be supported in its shipping container. Relative motion between the test surface and the product shall not be permitted [7] [8].

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[5] [1] Brandenburg and Lee: Fundamentals of Packaging Dynamics, 2nd Edition, MTS Corporation, Minneapolis, USA, 1985

2.2. Experiment – Determining the Natural Freqluency of an Internal Product Component (Critical Element)

2.2.1. Description of Testing Product Model

Actual model of the product that has been tested is shown in Figure 4. As described in section 1.4.3. (Vibrations of Packaged Product) of this work, and as shown in Figures 2 and 3, the physical model of the product that was tested had a critical element. That critical element (cantilever beam) made of Plexiglas was the most fragile component of the product model. It was the part that was most sensitive to damage by vibrations.

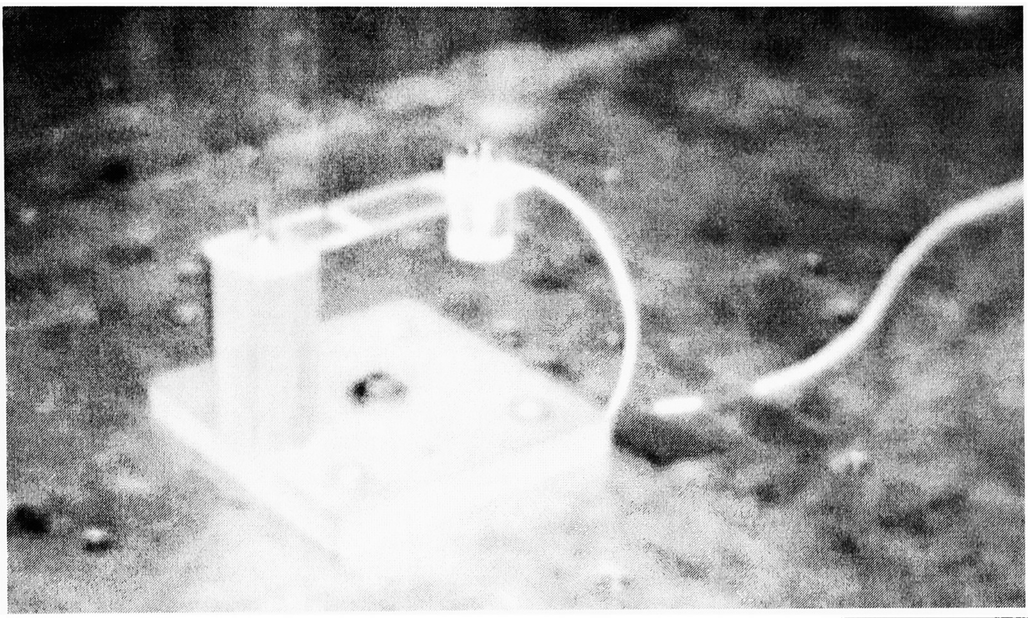


Figure 4. Model of the Product That Has Been Tested

Product model was made of wood and Plexiglas; Base – Oak, Post (support) – Pine and Clamped beam (cantilever) with clamped weight (internal component, critical element) – Plexiglas.

The model of the product has been fixed securely to the horizontal test surface by a screw. The test surface has been driven to move only vertically along a single axis in a sinusoidal motion. Relative motion between the test surface and the product is not permitted.

Figure 5 shows Product Model developed with I-DEAS software (SDRC Inc., USA)

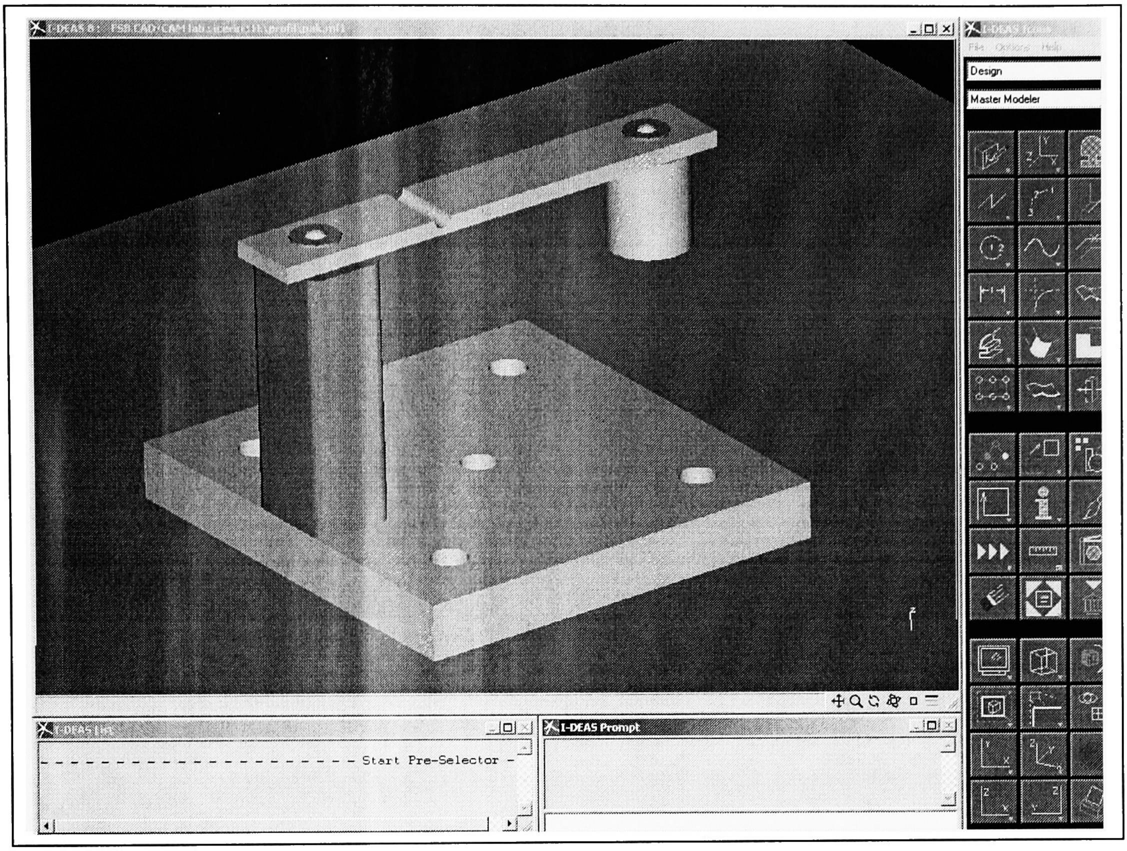


Figure 5. Testing Product Model – Model developed with I-DEAS software

In Figures 6a and 6b physical dimensions of the product model are shown (front & top view)

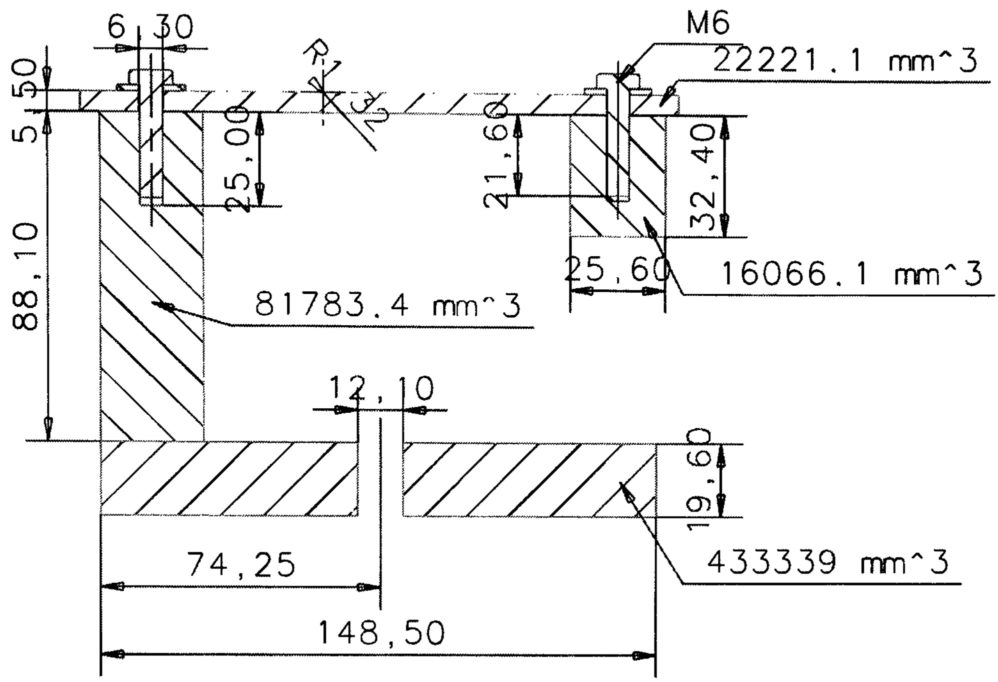


Figure 6a. Physical Dimensions of the Product Model (front view)

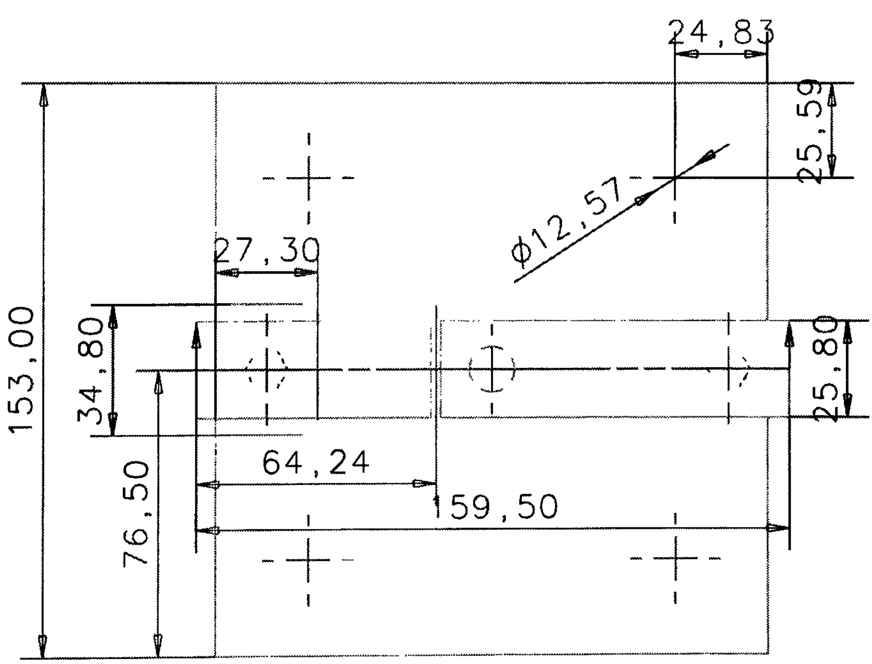


Figure 6b. Physical Dimensions of the Product Model (top view)

The Product System Model reduced to its simplest form is shown in Figure 7. That model may be further reduced to something that we may investigate mathematically.

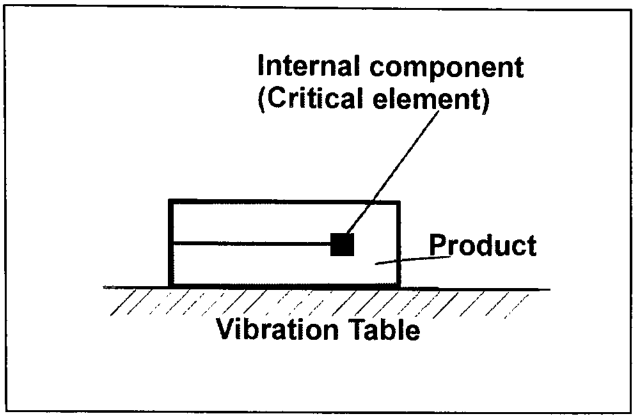


Figure 7. The Reduced Product System Model

2.3. Testing Machine at the R.I.T.

Vibration Machine Test System at the R.I.T. is Lansmont model 7000-10 (Lansmont Corporation, Monterey, CA, USA). Characteristics[6] of the vibration machine test system are shown in Appendix 1.

6 Instruction manual for the Lansmont model 7000-10, Touchtest Vibration System; Lansmont Corporation, Ryan ranch research park, 5 Harris Court, Building N, Monterey, CA 93940-5739

2.4. Experiment

The experiment has been performed on a Lansmont model 7000-10 vibration test machine. The frequency and amplitude of motion were variable and under control. The product (model) tested vibrated at acceleration and frequency levels that determine if the product resonance exists in the expected frequency range. The frequencies ranged from 3 to 100 Hz. Acceleration level to excite resonance was constant, 0.5 g.

The natural frequency of a cantilever beam is identified using a Lansmont vibration table capable of producing sinusoidal motion. The product (model) was securely attached to the table, and the system was vibrating at frequencies between 3 and 100 Hz with an associated maximum acceleration of 0.5 g (a constant 0.5 g's is commonly employed throughout the frequency range sweep).

2.4.1. Experiment Results (Natural Frequency)

Natural frequency of the clamped beam (cantilever) with clamped weight – all made of Plexiglas – obtained on vibration table was **26.777 Hz**. It can be seen on result diagrams, Figure 8 (Part 1 and Part 2).

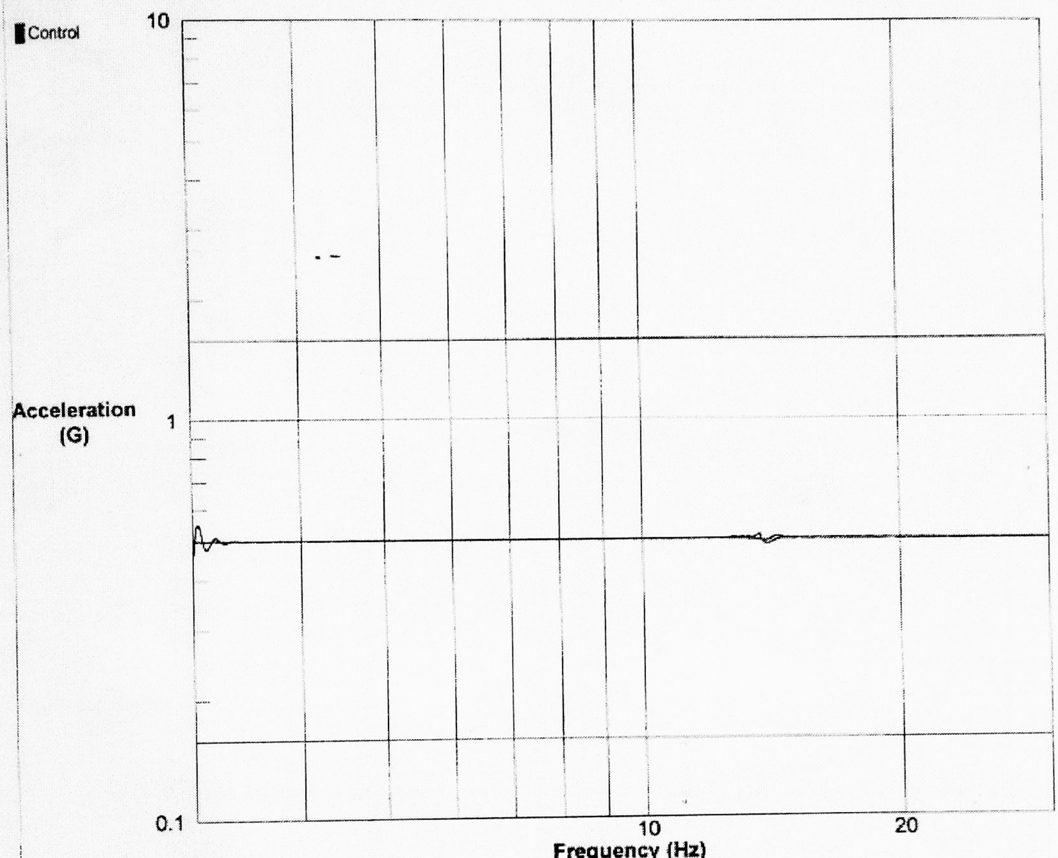
It should be noted that some of the factors affected accuracy. Those factors include: the fixturing and mounting methods for the accelerometers, and relative motion between the test surface and the product (model) tested (it is not permitted).

Control & Input Data	Ortho Clinical Data has not yet been saved	1:44:41 PM Jun-26-2000

Lansmont Corporation — TouchTest Vibration

TestQ ID: Default **Control:** 0.50 G pk **Elapsed:** 0 hr 6 min 38 sec **Profile:** 1 of 1
Profile ID: 3 to 100 Hz 0.5 G 2 oct per min 2 **Demand:** 3.0 Hz **Remaining:** 0 hr 0 min 0 sec **Total:** 0 hr 6 min 38 sec

Marker Freq (Hz)	Accel Up (G)	Accel Dn (G)	Vel Up (In/S)	Vel Dn (In/S)	Disp Up (In da)	Disp Dn (In da)



Lot 750 and Lot 790 0 degree shipping test

Figure 8 (Part 1). Result Diagrams for Model Testing at the R.I.T. Laboratory

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Lansmont Corporation — TouchTest Vibration

TestQ ID: Default
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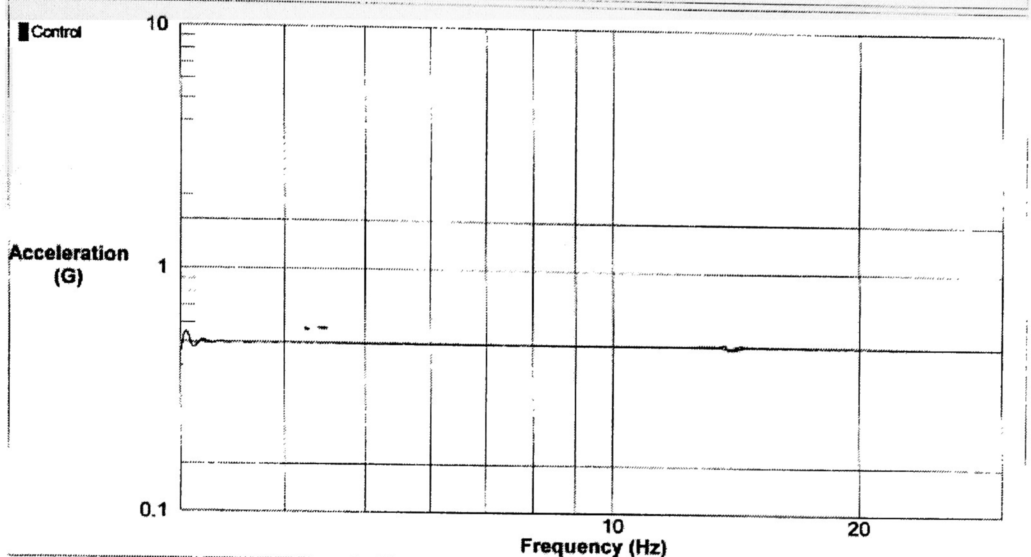
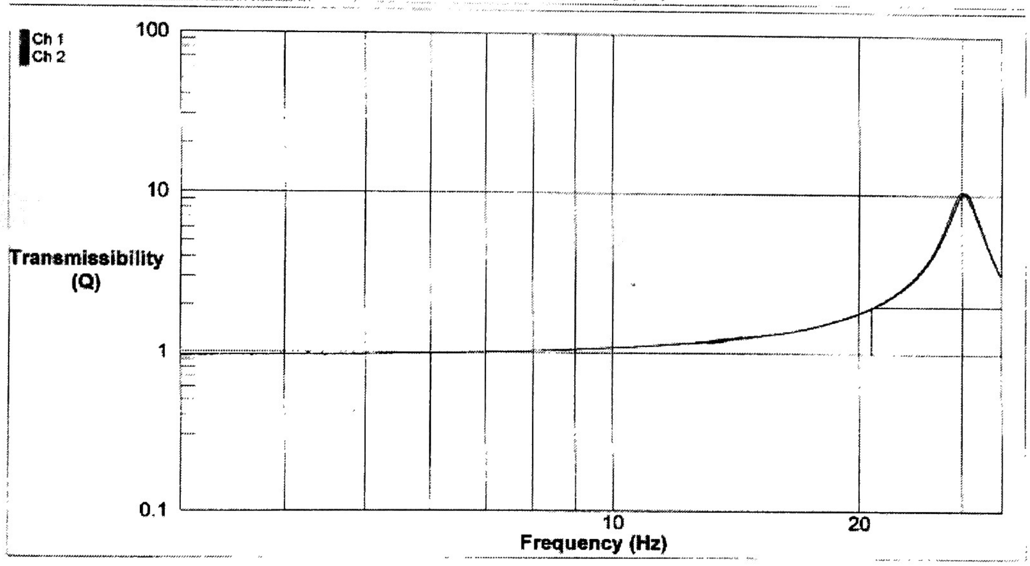
Demand: 3.0 Hz
Elapsed: 0 hr 6 min 38 sec
Remaining: 0 hr 0 min 0 sec
Profile: 1 of 1
Total: 0 hr 6 min 38 sec





Summary - Trans — Ortho Clinical
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Lansmont Corporation — TouchTest Vibration

	Channel Description:	Fn (Hz)	Trans Up (Q)	Trans Dn (Q)	Lo Limit (Hz)	Hi Limit (Hz)
Ch 2	Channel 2	26.777	10.11	10.49	20.749	30.061

Figure 8 (Part 2). Result Diagrams for Model Testing at the R.I.T. Laboratory

3. Bond Graphs Based Object Oriented Approach to Determination of Natural Frequency

This section deals with the development of the Bond Graph model of the product model described in section 2.2.1. (Description of Testing Product Model). Based on that model, predictions of the dynamic behavior and the vibration characteristics of the model subjected to vibrations based on the Bond Graph methodology could be obtained.

In this work the approach given in reference [2] was followed.

3.1. Model Representation

The solution of the problem is based on a *model* (a model of a system). For that purpose the system is simplified by abstracting its observable attributes. Attention is usually paid only to the features that are relevant to the problem being solved. Based on such an idealised picture, the system is described in a suitable form, and this is used as a basis for the solution of the problem.

Simulation models are used to simulate the behaviour of engineering products and processes in their environment. By experimenting on models instead of real equipment, its behaviour can be studied even before it is built and purchased. Consequently, Simulation models can be used at various stages of design - from the early stages of conceptual design to the final prototype testing.

Modeling and simulation play an important role in the design of such products. The concept of *System* plays a central role when building a model for a particular problem. We pay attention only to that part which we are interested in. Such part is termed the system for the given problem. Everything that is not included in the system constitutes its environment

Further model development is more concerned with the system than with its environment. We usually speak of a model of a system, but it is understood that there is an environment, which the system interacts with. These interactions are relatively simple and can be described by actions at the boundary of the system.

The system usually consists of *components*. Decomposition of a complex system into its components generally simplifies the modeling task and gives a better insight into the structure of the system. When doing that it is very useful to retain the structure of the system in the model too.

If system decomposition is to be continued further, every component should be treated as another system consisting of simpler components. At each level of decomposition the model of the corresponding component can be represented as compounded, i.e., consisting of simpler component models. Finally a level is reached where components are treated as *elementary* ones, i.e., not consisting of further simpler components. Such components are modeled as entities and define the level of detail of the modeling process.

It should be pointed out that it is not necessary that the model structure correspond to that of the physical system in all details. We usually start off with relatively crude models, and details are added later by elaborating the models of their components.

Top-down system decomposition and bottom-up composition are powerful modeling techniques. To use their full power it is necessary do define how to model components that we treat as entities (elementary components) and how to describe interactions between them. In engineering it is natural to base these considerations on physical reasoning using knowledge from various branches of physics. Such an approach is sometimes termed *physical modeling* [9].

Processes taking place in engineering systems generally belong to various physical domains, e.g., rigid and solid body mechanics, fluid mechanics, electricity and magnetism, semiconductor physics, thermodynamics and others. It means that we have to deal with multi-domain physical processes. One well-known approach in engineering designed to deal with such multi-domain physical problems is the Bond graph method of Henry Paynter.

The Bond graph method uses the effort-flow analogy to describe physical processes [10,11]. Processes are represented graphically in a form of elementary components (bond graph elements) with one or more *ports*.

The process "seen" at a port is described by a pair of *effort* and *flow* variables the product of which is power. Such variables are called *power variables*. There is flow of power through every port in or out of the component. The direction of the power flow is depicted by a *half-arrow*. In addition to power variables there are also variables that represent the accumulation of efforts and of flows over time. These variables are termed *generalized momenta* and *generalized displacements* respectively.

The bond graph shows the structure of the model in a way that resembles the structure of a real system.

It should be stressed that system decomposition combined with the bond graph modeling method leads to a lumped-parameter model representation. For the case where variables inside a component change continuously over some area of space it is necessary to discretize them, i.e., to represent them by a finite number of parts (components). This can be done in various ways, e.g., by using the finite element discretization method.

3.2. Simulation Software

There are many software tools using bond graphs which support simulation, e.g., ENPORT (Rosenberg ,1974), TUTSIM (Kraan, 1974), CAMP (Granada, 1985), BONDYN (Felez et al, 1990), MMT toolbox (Gawthorp, 1996). Of these CAMAS (Broenink et all, 1995) seems to be more relevant to the approach used in this work. It supports bond graph and block diagrams representation in multidimensional form and hierarchical modeling. CAMAS consists of different modules as separate programs. It normally runs on workstations, but there is also a restricted version for PCs (no multidimensional elements and no implicit integration methods) [2].

The ***BondSim***© system has been used for problem solving described in this work. It is designed as a compact modeling framework based on the object oriented approach in a visual environment. The core of the modeling system's philosophy is system decomposition and composition. This is based on a general model of the component, which contains all information on its structure, properties, and behavior. The model of a system is represented as a tree of interconnected component models that are held in the library. The modeling system is designed in a visual environment which supports graphical editing and an automatic generation of the model's database. The functional model representation is based on bond graph methodology. Much attention is devoted to describing standard physical processes in a symbolical form. In addition to basic bond graph elements, a block diagram-like description is also supported. The final system model is generated in a symbolical form. Simulation of the system simulator is based on solving differential algebraic equation using suitable methods [2].

The modeling and simulation environment were designed as a compact computer workbench BondSim© [2]. All the bond graph diagrams in this work were created using the BondSim©[7]. It was also used for solving the natural frequency problem described in this work.

3.3. Bond Graphs Model of Testing Element

3.3.1. BG Model

In order to develop the model we will analyze physical situation shown in Figure 9 first. We can identify several components: testing machine, package (i.e., the model with its components exposed to vibrations and motion). Physical interaction happens between the testing machine and the package (model).

The model of the testing machine is set up to simulate forced vibrations imposed on the package system (from outside the package system). It is modeled as a sine function exciter with a changeable frequency.

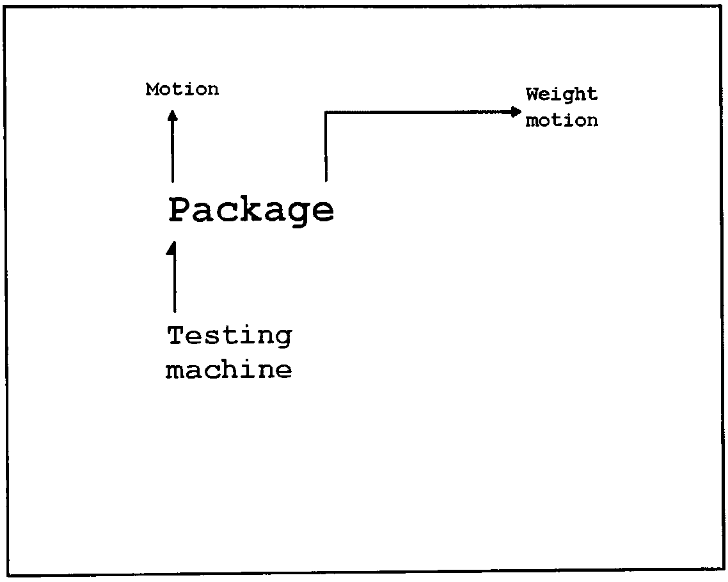

Figure 9. Bond Graph Package system model and environment

The package (testing model) vibrates as a whole unit, and the information on the motion, i.e., displacement of the package unit is thus obtained.

Velocity is changed pursuant to the sine function $v_p = v_0 \sin(\omega t)$, where v_0 is an amplitude and ω is a circular frequency defined as $\omega = 2\pi f$. Displacement is obtained by the integration $x_p = \int_0^t v_p dt$. This provides us with the information on motion (Figure 9)

[7] The BondSim supports, for documentation purposes, printing of screen images into a file in EMF (Enhanced Windows Metafile) format, which is supported by main word or graphic programs including MS Word, Corel Draw etc.

In the BondSim© program each real component is represented by its name which serves as its *word model.* The program really creates an empty object in the computer memory serving as a model of a real component. We call this memory object simply a *component.* The component is depicted on the computer screen by a suitable text that identifies it. Our package (testing model) consists of three (3) such components named Body, Package Plate (i.e., the console) and Block (Figure 10). The Package Plate (console) is fixed to the body by a screw and it carries the weight (Block) on the other side (its right side). Block vibrations in relation to Body are taken into consideration and are sent to display, as seen in the diagram (Figure 9), as the Weight motion.

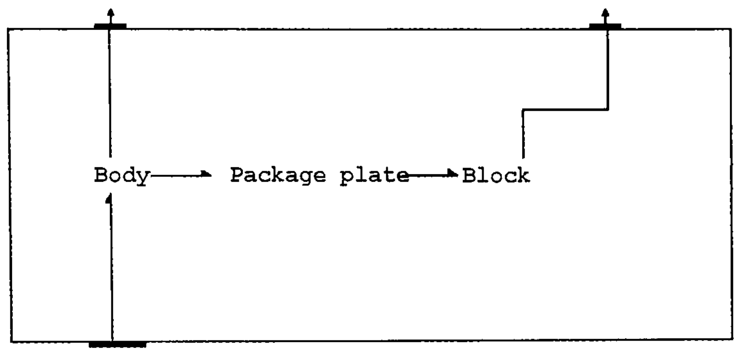

Figure 10. Bond Graph Package system (testing model) decomposition (components and environment)

The Package Plate (cantilever) itself is divided into four (4) sections (hyper-elements) as shown in Figure 11. Each of these sections consists of five (5) finite elements submitted in the form of the Bond Graphs Finite Elements (BFE) shown in Figure 12. The BFE detailed structure is shown in Figure 13. Explanation of the structure shown in Figure 13 is further elaborated in section (3.4. Mathematical model of the beam vibration, clause 3.4.4. The beam dynamics) of this work.

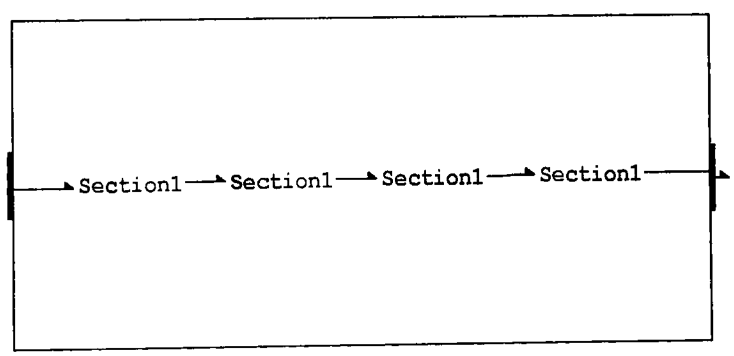

Figure 11. Bond Graph model of Package Plate (cantilever) division (partition)

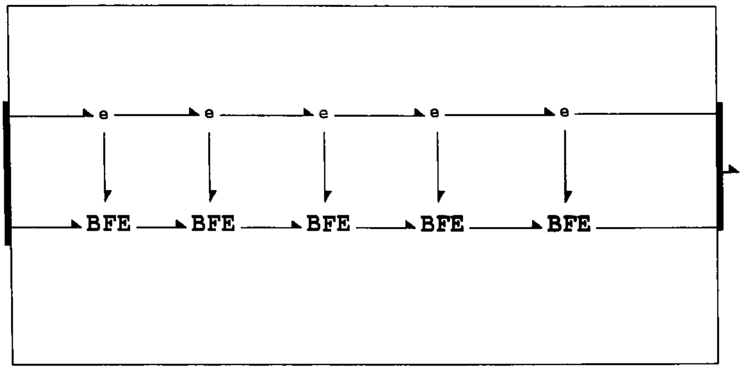

Figure 12. Bond Graph Finite Elements (BFE)

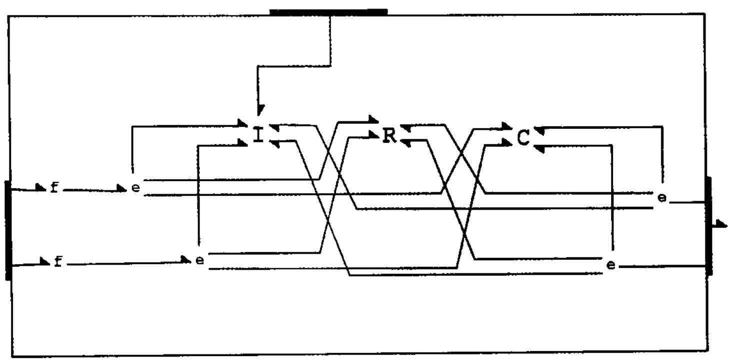

Figure 13. Bond Graph beam element BFE

Block element (component) represented as a dynamic model is shown in Figure 14, and its detailed structure is shown in Figure 15.

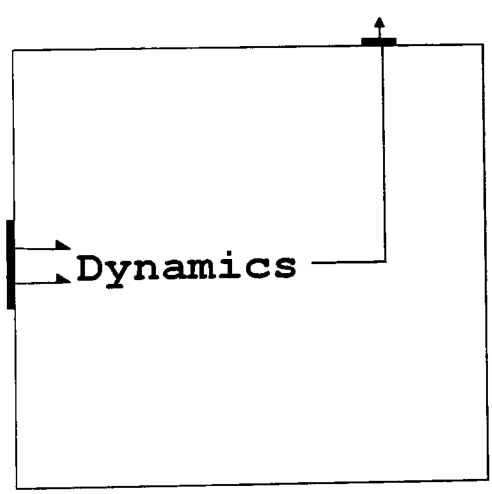

Figure 14. Bond Graph Block element as a Dynamic model

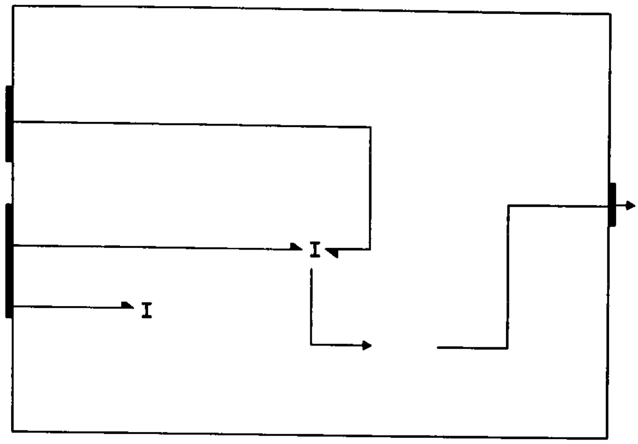

Figure 15. Detailed Structure of the Bond Graph Dynamic model of the Block element

Full explanation of the structure shown in Figure 15, is given further in the section (3.4. Mathematical model of the beam vibration, clause 3.4.5. The weight dynamics) of this work.

In that way the model is formed like a multi-level structure that corresponds to the structure of the physical system (the model that was tested on a vibration table at the R.I.T. Lab).

3.3.2. Mechanical Properties of Testing Element Material

Mechanical properties of Testing Element material were taken from PLASPEC™ Materials Selection Database [12], and also from [13] and [14], [15], [16].
Complete list of Mechanical properties of Plexiglas (PMMA - Polimetilmetaacrylat) according to [12] is given in Appendix 2.

For our simulation we will need **Modulus of Elasticity (E)**. For Plexiglas (PMMA) it is in the range of 400,000 to 500,000 (p.s.i.). For our simulation we will calculate with mean (average) value of 450,000 (p.s.i.)

E = 450,000 (p.s.i.) x 6,895 = **3102750 (kN/m^2) = 3102,75 (N/mm^2)**

The product model consists of a beam with weight and support, and is made of wood and Plexiglas.

In our case the base and the post (i.e., the support), which are made of wood, are not relevant for our calculation of the natural frequency as we consider that our critical element (the beam) is clamped at the end at which there is the post.

By taking the volume data from Figure 6a we can calculate respective masses of the console (M1) and the weight (M2) as we need them for our simulation.

Specific mass (Density), ρ, of Plexiglas is [11]: ρ = 1,18 g/cm^3 = 1180 kg/m^3

Mass of critical part, beam (Plexiglas) - M1= 0,02622 kg:
M1 = V1 x ρ = 2222,1 (mm^3) x 1,18 (g/cm^3) x 10^{-3} = 26,22 g = 0,02622 kg

Mass of Weight (Plexiglas cylinder + screw) – M2 = 0,03117 kg
M2.1 = V2 x ρ = 16066,1 (mm^3) x 1,18 (g/cm^3) x 10^{-3} = 18,96 g = 0,01896 kg
M2.2 = V_{screw} + $V_{screw\text{-}pad}$ = 1193,92 mm^3 + 361,911 mm^3 = 1555,831 mm^3
ρ steel = 7850 kg/m^3

M2.2 = 1555,831 mm^3 x 10-9 x 7850 (kg/m^3) = 0,001555831 x 7850 = 0,01221 kg
M2 = M2.1 + M2.2 = 0,01896 + 0,01221 = 0,03117 kg

Table 2. Mechanical properties of Plexiglas (PMMA) and physical properties of testing model elements

	Specific Mass (Density), ρ	**Modulus of Elasticity (E)**	**Mass of critical part, console (Plexiglas)**	**Mass of Weight (Plexiglas cylinder + screw)**
Plexiglas (PMMA - Polimetilmetaacrylat)	**ρ = 1,18 g/cm^3 = 1180 kg/m^3**	**3102750 (kN/m^2) = 3102,75 (N/mm^2)**	**0,02622 kg**	**0,03117 kg**

3.4. Mathematical Model of the Beam Vibrations

3.4.1. Introduction

The packaging system consists of a frame to which a cantilever beam carrying a weight is attached (Figure 16). The system was fixed to the table of a testing machine that generates harmonic vibrations in the vertical direction. When vibrations are applied the cantilever and the attached weight vibrates. In order to develop a bond graph model for simulating frequency response of the weight, it is necessary to develop governing kinematical and dynamical relations for the beam and the attached weight. The beam was modeled using the finite element technique and the Euler-Bernoully beam theory [17].

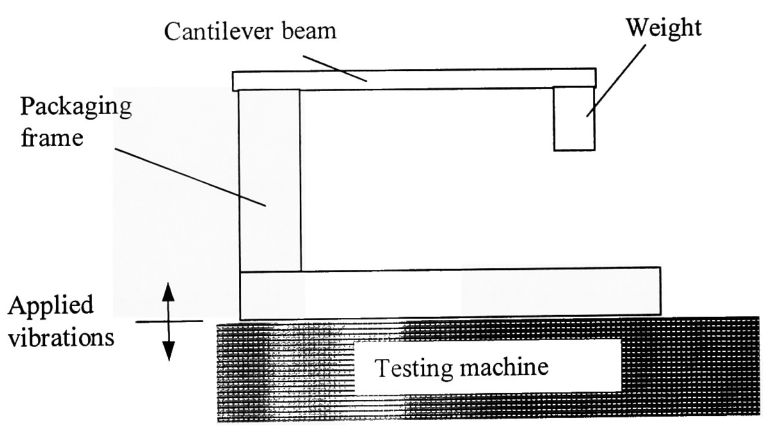

Figure 16. The packaging system subjected to vibration testing

3.4.2. Basic kinematics

The beam was treated as an assemblage of finite elements. In order to describe the motion of the beam several co-ordinate systems were defined (Figure 17). We will use an approach similar to the floating frame approach of Shabana (1998) [18].

The motion of a complete system is described with a reference to the co-ordinate system $O_r x_r z_r$ fixed to the ground (the testing machine body). Next a body co-ordinate system $O_b x_b z_b$ was defined that moves with the package frame. Because the system moves in the vertical direction only, the axes of the body frame were assumed to be parallel to the axes of the reference frame. The element co-ordinate systems $O_i x_i z_i$ (i = 1,..,n) were defined as attached to each of the finite elements. The axes of the element co-ordinate system were initially assumed to be parallel to the body co-ordinate system. Also a co-ordinate system $O_w x_w z_w$ was defined, the axes of which are parallel to the body axes, and their origin is attached to the point of attachment of the weight to the beam.

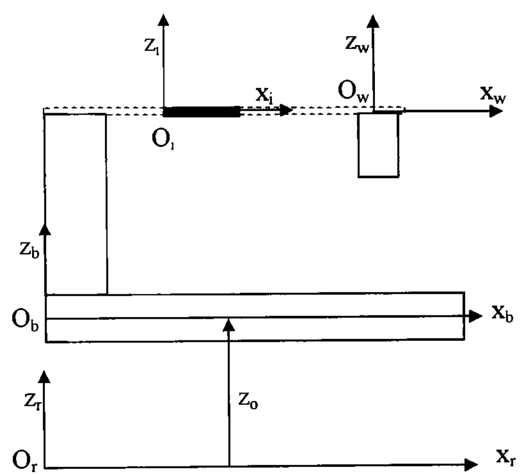

Figure 17. Co-ordinate systems of the package

The motion of the body co-ordinate system is described by the co-ordinate z_0 given by

$$z_o = A\cos(2\pi ft) + A_0 \tag{1}$$

where A is the amplitude of vibrations generated by the testing machine and f its frequency and A_0 is a DC value.

The motion of the beam element "i" in respect to the body co-ordinate system was described by two end vertical displacements w_{i1} and w_{i2}, and two end slopes θ_{i1} and θ_{i2} (Figure 18).

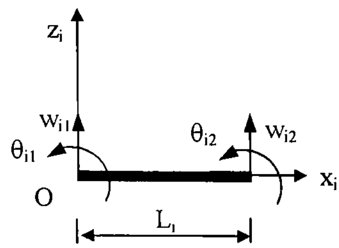

Figure 18. Element co-ordinates

The set of generalized co-ordinates of the element consists of five co-ordinates. These co-ordinates can be represented in a matrix form such as

$$\mathbf{q}_i = \begin{pmatrix} z_0 \\ \mathbf{q}_{ei} \end{pmatrix} \tag{2}$$

where

$$\mathbf{q}_{ei} = \begin{pmatrix} w_{i1} \\ \theta_{i1} \\ w_{i2} \\ \theta_{i2} \end{pmatrix} \tag{3}$$

Vertical displacement of the beam with respect to reference co-ordinate system is given by

$$z_i = z_0 + w_i \tag{4}$$

where w_i is the displacement of the beam in respect to the body co-ordinate system. This last displacement can be expressed as (Cook et al (1989) [19].)

$$w_i = \mathbf{S}\mathbf{q}_{ei} \tag{5}$$

where **S** is the matrix of the shape functions

$$\mathbf{S} = \begin{pmatrix} S_1 & S_2 & S_3 & S_4 \end{pmatrix} \tag{6}$$

with

$$\left.\begin{aligned} S_1 &= 1 - 3\xi^2 + 2\xi^3 \\ S_2 &= L\left(\xi - 2\xi^2 + \xi^3\right) \\ S_3 &= 3\xi \ - 2\xi^3 \\ S_4 &= L\left(-\xi^2 + \xi^3\right) \end{aligned}\right\} \tag{7}$$

where $\xi = x_i/L_i$. From Eqs. (4) and (5) we have

$$z_i = z_0 + \mathbf{S}\mathbf{q}_{ei} \tag{8}$$

Corresponding velocities are

$$v_i = v_0 + \mathbf{S}\mathbf{v}_{ei} \tag{9}$$

where

$$v_0 = z_i \tag{10}$$

and

$$\mathbf{v}_{ei} = \dot{\mathbf{q}}_{ei} = \begin{pmatrix} w_{e1} \\ \theta_{e1} \\ w_{e2} \\ \theta_{e2} \end{pmatrix} \tag{11}$$

A similar approach is used for weight motion. The generalized co-ordinate for the weight is

$$\mathbf{q}_w = \begin{pmatrix} z_0 \\ z_{wb} \\ \theta_{wb} \end{pmatrix} \tag{12}$$

where z_{wb} and θ_{wb} are the vertical translation and rotation of the weight co-ordinate system in respect to the package body coordinate system (Figure 19). Corresponding velocities are

$$\dot{\mathbf{q}}_w = \begin{pmatrix} v_0 \\ z_{wb} \\ \theta_{wb} \end{pmatrix} \tag{13}$$

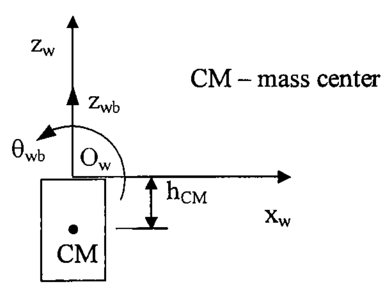

Figure 19. Coordinates of the weight

The components of the center of the mass velocity in the reference coordinate system are

$$\left.\begin{array}{l} v_{CMx} = h_{CM}\,\theta_{wb} \\ v_{CMz} = v_0 + z_{wb} \end{array}\right\} \tag{14}$$

3.4.3. Lagrange equations of element motion

In order to develop the equation of the beam vibration the Lagrange equation of a beam element motion was used. These equations read

$$\frac{d}{dt}\left(\frac{\partial T_i}{\partial \dot{q}_i}\right) - \left(\frac{\partial T_i}{\partial q_i}\right) = \mathbf{Q}_i \tag{15}$$

where T_i is the kinetic energy of the i-th element being the vector of generalized forces.

The generalized forces include effects of the element's elastic forces as well as the forces corresponding to generalized velocities, i.e., the nodal forces and moments. The effects of gravity forces were neglected, as a common praxis in mechanical vibration analysis. The generalized forces now read (Shabana (1998) [18])

$$\mathbf{Q}_i = -\begin{pmatrix} 0 & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_i \end{pmatrix}\begin{pmatrix} z_0 \\ q_{ei} \end{pmatrix} + \begin{pmatrix} F_{0i} \\ \mathbf{Q}_{ei} \end{pmatrix} \tag{16}$$

where $\mathbf{K}_i$ is the stiffness matrix and F_{0i} and $\mathbf{Q}_{ei}$ are the nodal forces.

Assuming a straight beam of constant cross section the stiffness matrix is given by (Cook et all (1989) [19])

$$\mathbf{K}_i = \frac{E_i I_i}{L_i^3}\begin{pmatrix} 12 & 6L_i & -12 & 6L_i \\ 6L_i & 4L_i^2 & -6L_i & 2L_i^2 \\ -12 & -6L_i & 12 & -6L_i \\ 6L_i & 2L_i^2 & -6L_i & 4L_i^2 \end{pmatrix} \tag{17}$$

where E_i is the Young's modulus (Modulus of Elasticity), I_i is the section moment of inertia, and L_i is the i-th element length.

The vector of generalized forces $\mathbf{Q}_{ei}$ consists of forces and the moment acting at beam nodes, i.e.

$$\mathbf{Q}_{ei} = \begin{pmatrix} F_{e1} \\ M_{e1} \\ F_{e2} \\ M_{e2} \end{pmatrix} \tag{18}$$

After substitution Eq. (16) in Eq. (16) we get

$$\frac{d}{dt}\left(\frac{\partial T_i}{\partial \dot{q}_i}\right) - \left(\frac{\partial T_i}{\partial q_i}\right) + \begin{pmatrix} 0 & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_i \end{pmatrix}\begin{pmatrix} z_0 \\ q_{ei} \end{pmatrix} = \begin{pmatrix} F_{0i} \\ \mathbf{Q}_{ei} \end{pmatrix} \tag{19}$$

In order to complete the equation of motion it is necessary to evaluate kinetic energy and to find out corresponding partial derivatives.

3.4.4. The beam element dynamics

The kinetic energy of a beam element is given by

$$T_i = \frac{1}{2}\int_0^{L_i} \rho A_i v_i^2 dx \tag{20}$$

where ρ is mass density of the beam, and A_i is element cross section area. Using Eq. (9) we get

$$v_i^2 = (v_0 + \mathbf{v}_{ei}^T \mathbf{S}^T)(v_0 + \mathbf{S}\mathbf{v}_{ei}) = v_0^2 + 2v_0 \mathbf{S}\mathbf{v}_{ei} + \mathbf{v}_{ei}^T \mathbf{S}^T \mathbf{S}\mathbf{v}_{ei}$$

Substituting in Eq. (20) we have

$$T_i = \frac{1}{2} m_i v_0^2 + m_i v_0 \int_0^1 \mathbf{S} d\xi \;\; \mathbf{v}_{ei} + \frac{1}{2} m_i \mathbf{v}_{ei}^T \int_0^1 \mathbf{S}^T \mathbf{S} d\xi \;\; \mathbf{v}_{ei}$$

where $m_i = \rho A_i L_i$ is the element mass. The kinetic energy expression can also be written as

$$T_i = \frac{1}{2} m_i v_0^2 + v_0 \mathbf{N}_i \mathbf{v}_{ei} + \frac{1}{2} \mathbf{v}_{ei}^T \mathbf{M}_i \mathbf{v}_{ei} \tag{21}$$

where

$$\mathbf{N}_i = m_i \int_0^1 \mathbf{S} d\xi \tag{22}$$

and

$$\mathbf{M}_i = m_i \int_0^1 \mathbf{S}^T \mathbf{S} d\xi \tag{23}$$

The last matrix is known as the consistent mass matrix (Cook et al (1989) [19]). Using shape functions of Eq. (7) we get

$$\mathbf{N}_i = \frac{m_i}{12}(6 \quad L_i \quad 6 \quad -L_i) \tag{24}$$

and

$$\mathbf{M}_i = \frac{m_i}{420}\begin{pmatrix} 156 & 22L_i & 54 & -13L_i \\ 22L_i & 4L_i^2 & 13L_i & -3L_i^2 \\ 54 & 13L_i & 156 & -22L_i \\ -13L_i & -3L_i^2 & -22L_i & 4L_i^2 \end{pmatrix} \tag{25}$$

Next we will define the momentum vector by

$$\mathbf{p}_i = \frac{\partial T_i}{\partial \dot{\mathbf{q}}_i} \tag{26}$$

From Eq. (21) we get

$$\mathbf{p}_i = \begin{pmatrix} m_i & \mathbf{N}_i \\ \mathbf{N}_i^T & \mathbf{M}_i \end{pmatrix}\begin{pmatrix} v_0 \\ \mathbf{v}_{ei} \end{pmatrix} \tag{27}$$

Using the momentum, the first term on the right side of Eq. (15) reads

$$\frac{d}{dt}\left(\frac{\partial T_i}{\partial \dot{q}_i}\right) = \dot{\mathbf{p}}_i$$

The second term is equal to zero. Thus the equation of motion of a beam element can be written as

$$\frac{d\mathbf{p}_i}{dt} + \begin{pmatrix} 0 & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_i \end{pmatrix} \begin{pmatrix} z_0 \\ q_{ei} \end{pmatrix} = \begin{pmatrix} F_{0i} \\ \mathbf{Q}_{ei} \end{pmatrix} \tag{28}$$

We will further modify this equation by introducing beam damping

$$\frac{d\mathbf{p}_i}{dt} + \begin{pmatrix} 0 & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_i \end{pmatrix} \begin{pmatrix} v_0 \\ \mathbf{v}_{ei} \end{pmatrix} + \begin{pmatrix} 0 & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_i \end{pmatrix} \begin{pmatrix} z_0 \\ q_{ei} \end{pmatrix} = \begin{pmatrix} F_{0i} \\ \mathbf{Q}_{ei} \end{pmatrix} \tag{29}$$

The beam damping matrix $\mathbf{R}_i$ is defined by relation

$$\mathbf{R}_i = \alpha \mathbf{M}_i + \beta \mathbf{K}_i \tag{30}$$

where α and β are suitable constants. This type of damping is known as Rayleigh damping (Cook et al (1989) [19]).

The equation of the element motion as given by Eqs. (27), (29) and (30) can be easily represented by bond graph element BFE shown in Figure 20. The element has two pair of external ports with associated effort/flow variables corresponding to generalized forces on the right side of Eq. (29), and to velocities given by Eqs. (10) and (11). The inertia of the beam element is represented by an inertial element **I**, which implements Eq. (27). The elasticity of the element is represented by a capacitive element **C** which implements the third term on the left side of Eq. (29). Finally, a resistive element R is used to describes Rayleigh type damping in the beam.

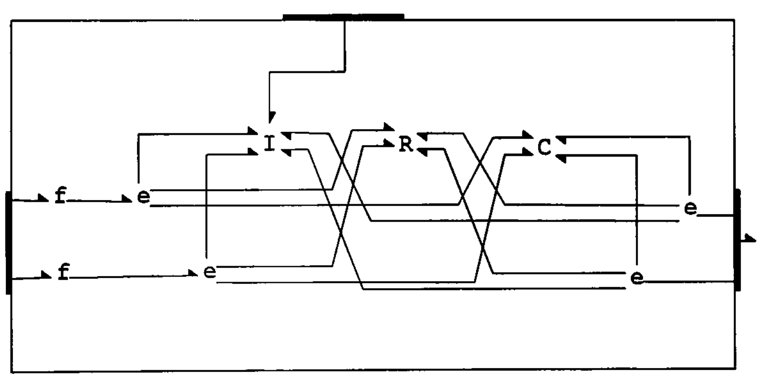

Figure 20. Bond graph beam element BFE

3.4.5. The weight dynamics

Kinetic energy of the weight is given by

$$T_w = \frac{1}{2} m_w v_{CM}^2 + \frac{1}{2} I_{CM} \dot{\theta}_{wb} \tag{31}$$

where m_w is the mass of the weight and I_{CM} is its mass moment of inertia in respect to the mass center. Using Eqs. (14) we get

$$T_w = \frac{1}{2} m_w \left[h_{CM}^2 \theta_{wb}^2 + (v_0 + z_{wb})^2 \right]^2 + \frac{1}{2} I_{CM} \theta_{wb} \tag{32}$$

The momentum of the weight is defined by

$$p_w = \frac{\partial T_w}{\partial \dot{q}_w} \tag{33}$$

Using Eq. (32) we get

$$p_w = \begin{pmatrix} m_w & m_w & 0 \\ m_w & m_w & 0 \\ 0 & 0 & I_w \end{pmatrix} \begin{pmatrix} v_0 \\ z_{wb} \\ \dot{\theta}_{wb} \end{pmatrix} \tag{34}$$

where

$$I_w = I_{CM} + m_w h_{CM}^2 \tag{35}$$

is the mass moment inertia of the weight around the origin of its coordinate system.

The equation of the weight motion reads

$$\dot{p}_w = \mathbf{Q}_w \tag{36}$$

where the vector of generalized forces $\mathbf{Q}_w$ consist of forces and a moment corresponding to generalized velocities, i.e.

$$\mathbf{Q}_w = \begin{pmatrix} F_{0w} \\ F_w \\ M_w \end{pmatrix} \tag{36}$$

Dynamics of the motion of the weight defined by Eqs. (34)-(36) can be represented by the bond graph component in Figure 21.The component has three external ports corresponding to the pair of generalized forces of Eq. (36), and velocities of Eq. (13). The inertia of the weight is represented by two inertial elements corresponding to the two first rows, and to the third row respectively of Eqs. (34) and (36). The integrator is used for generation of the vertical weight displacement.

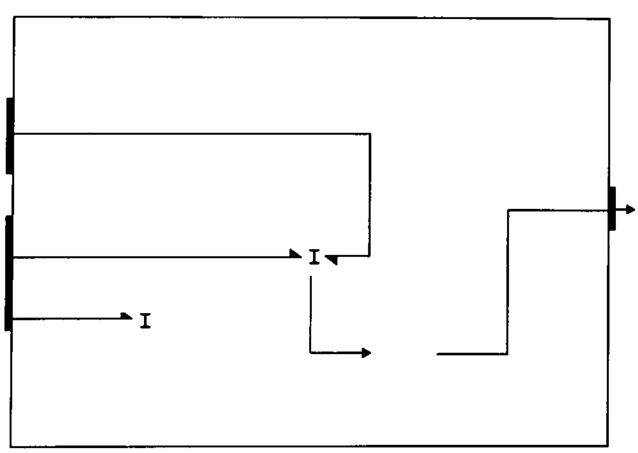

Figure 21. Bond graph model of dynamics of the weight

3.5. Executing Simulation

The ***BondSim©*** system has been used for problem solving described in this work [2]. It is designed as a compact modeling framework based on the object-oriented approach in a visual environment. The functional model representation is based on bond graph methodology. The model of the system is represented as three interconnected model components as explained earlier.

Physical parameters used are given in section 3.3.2. (Mechanical Properties of Testing Element Material). The model used is the one described in sections 2.2.1. (Description of Testing Product Model) and 3.3.1. (BG Model).

The final system model is generated in a symbolical form. Based on the model created the ***BondSim©*** produces the mathematical model in the form of a system of differential algebraic equations. Simulation of the system simulator is based on solving differential algebraic equation using suitable methods [2]. In that way the program generates required outputs (vibration amplitudes and amplitudes of Block vibrations against the Body). Those two (2) outputs have been generated for each frequency, and amplification spectrums were evaluated by using the Fast Fourier transformations.

Simulation was performed in a frequency spectrum of 1 Hz to 50 Hz.

Typical results are shown in Figs. (22 through 25) and they correspond to frequency of 33 Hz. In Figure 22 and 23 time diagrams of exciter input and of resulting weight motions are shown.

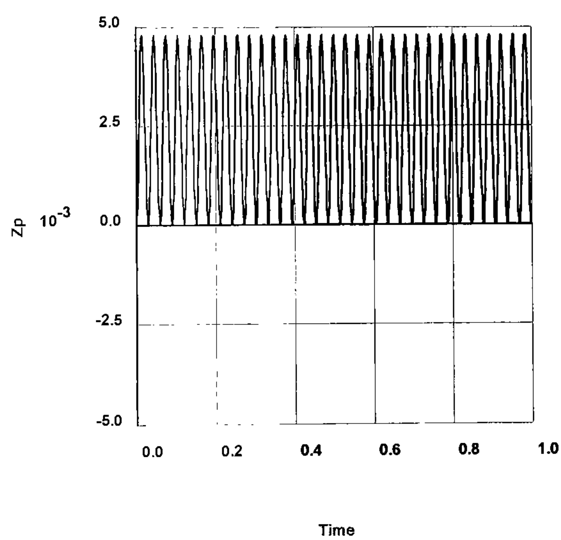

Figure 22. Input vibration of the package at 33 c/s

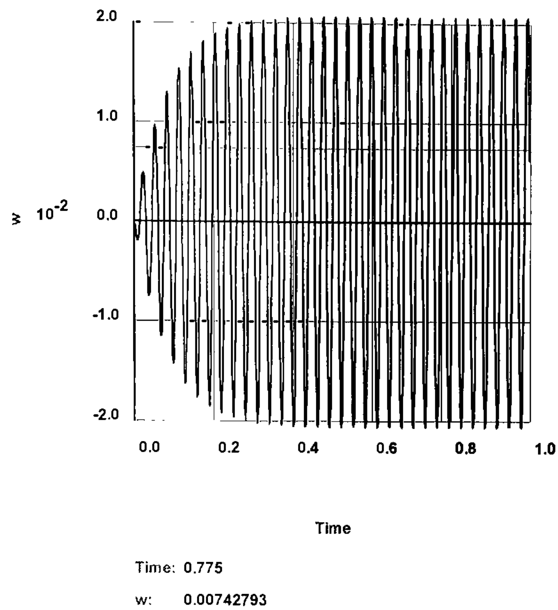

Figure 23. The response vibration of the weight attached to the cantilever beam end

In Figure 24 the corresponding frequency spectrum of input, and in Figure 25 the corresponding frequency spectrum of the weight motion are shown.

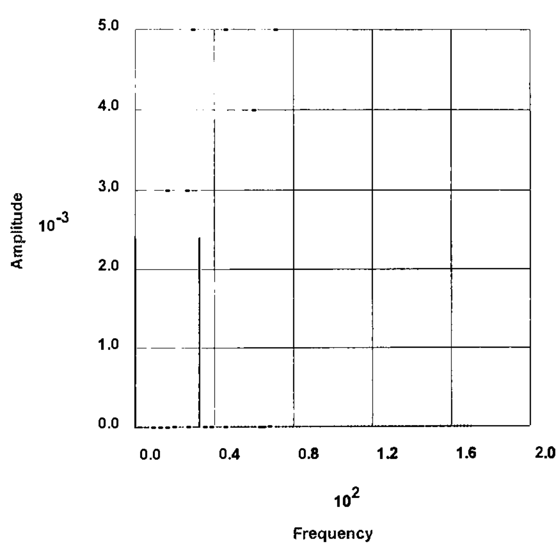

Figure 24. Frequency spectrum of input vibrations

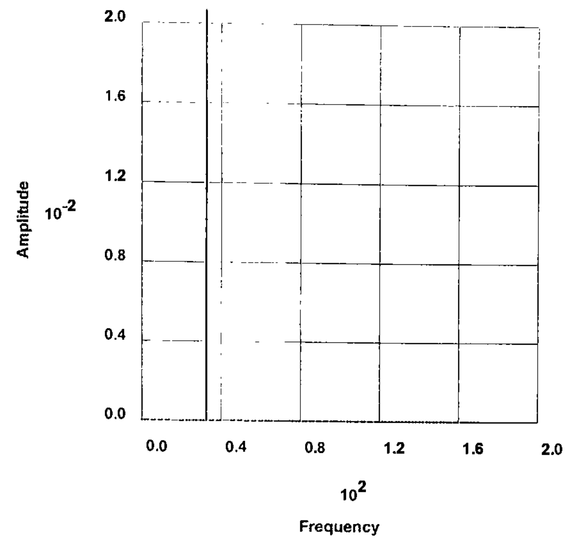

Figure 25. The frequency spectrum of the response

Based on the spectrum of input and output signals transmissibility (magnification factor) was calculated as $Q = 1 + (A_{output}/A_{input})$, where Q is transmissibility. It was done for a frequency range of 1 to 50 Hz and results are shown in Table 3 and in diagrams in Figure 26 and Figure 27, the latter in the log scale.

Table 3. Frequency response of the package plate, console (Rayleigh damping coefficients α = 20, $\beta = 5 \cdot 10^{-4}$)

Frequency [Hz]	Output amplitude A [m]	Input amplitude A [m]	Transmissibility Q
1	$7.316 \cdot 10^{-5}$	0.07958	1.001
10	0.0008051	0.007958	1.10
20	0.002311	0.003079	1.58
25	0.004262	0.003183	2.34
30	0.01126	0.002652	5.24
32	0.01911	0.002487	8.68
32.9	0.02077	0.002419	9.59
33	0.02069	0.002411	9.58
34	0.01765	0.002341	8.54
35	0.01380	0.002274	7.07
37.5	0.008197	0.002122	4.86
40	0.005753	0.001989	3.89
50	0.002726	0.001592	2.71

Note: Transmissibility $Q = 1 + (A_{output}/A_{input})$

As it can be seen, the **Natural** frequency obtained **by simulation is 32,9 Hz.**

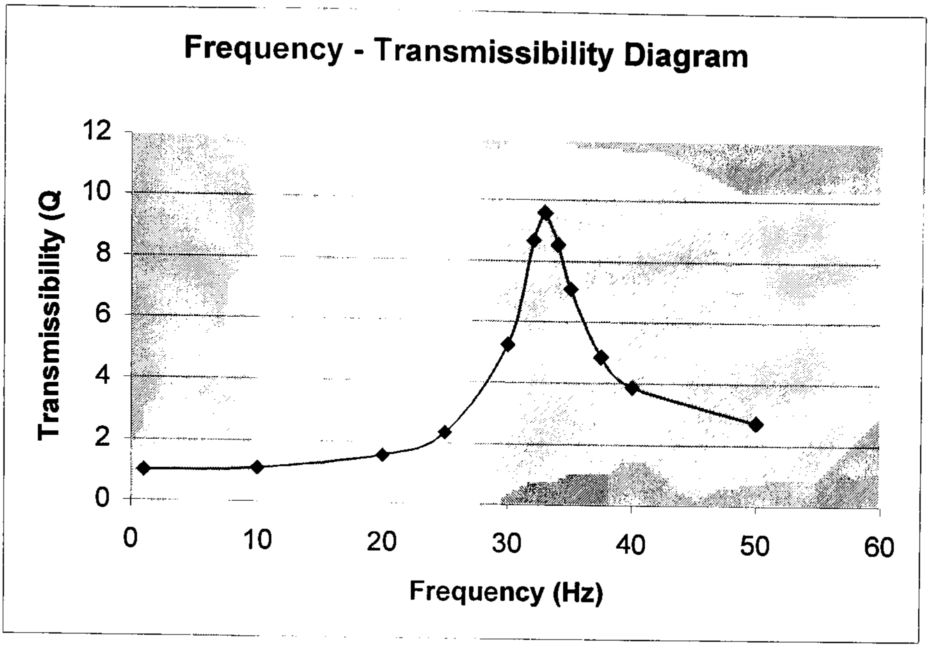

Figure 26. Results diagram Frequency - Transmissibility

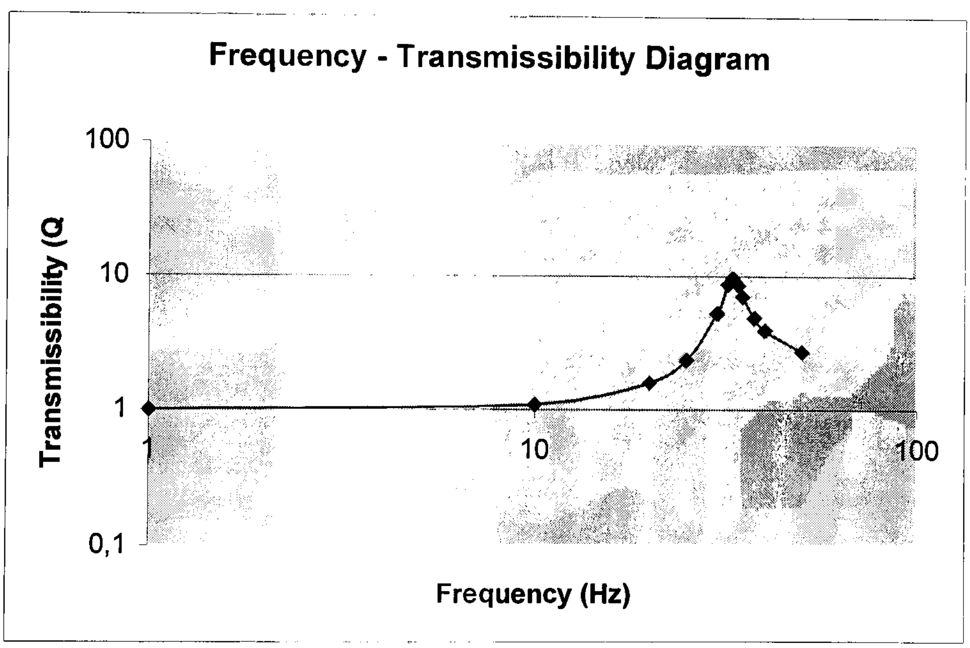

Figure 27. Results diagram Frequency – Transmissibility (Log scale)

Comparing the result (32,9 Hz) obtained by the BG simulation to the result obtained on the vibration table (26,777 Hz) it can be seen that they are close to each other.
It has to be emphasized and taken into consideration that the simulation was done using average (mean) values of parameters (mechanical properties and density) of Plexiglas (PMMA -

Polimetilmetaacrylat) as the exact values of the material used for the *beam* and *weight* were not known.

Besides, the simulation model applied was relatively simple (boundary conditions were simplified, and it would be necessary to apply a more detailed model of boundary conditions, as well as to model the groove in a more detailed way. Those features, as well as the fixturing of the beam and weight (clamping by screw), had an important implication for the result obtained by simulation). Nevertheless, it can be concluded that the results obtained are very positive, and that this work proves that the method applied here can be used to determine dynamic properties during the product design process (computer aided total Packaging distribution design process) before a specific product (part, component) is manufactured. In that way a need for intensive testing is reduced to those cases in which it is absolutely necessary (e.g., for security reasons, or if a hazard is involved)

4. Conclusion

This work has evaluated the application of Bond Graphs Based Object-Oriented Approach to Determination of Natural Frequencies of Packaging System Elements. Using the Bond Graphs, a detailed model of the Package-product system under testing was developed which closely simulated the real testing model and conditions. A model of testing equipment and packaging system was developed using the Bond Graph computer assisted modeling approach. Using simulation, vibrations characteristics of the packaging model were obtained, i.e., natural frequencies and other relevant properties of the system (model).
An analysis of the frequency characteristics was carried out by application of physical modeling using a suitable visual computer development platform. The results obtained by the simulation and those obtained on the vibration table (32,9 Hz vs. 26,777 Hz) were close to each other.

It has to be said that the simulation was done using average (mean) values of mechanical properties and density of Plexiglas (PMMA - Polimetilmetaacrylat) as the exact values of the material used for the *beam* and *weight* were not known.

Besides, the simulation model applied was relatively simple. Nevertheless, one may conclude that results were very positive, and that this work proves that method applied can be used to determine dynamic properties during the product design process (computer aided total Packaging distribution design process) before a specific product (part, component) is manufactured.

This work has also proved that the Bond Graphs based approach has its place in a computer aided total Packaging Distribution Design process. The entire design process should be speeded up by a continuous evaluation through simulation after each design/modification step, so that the finished package design meet the design objectives. It should also enable one to shorten the entire design/testing process, and insure that those expensive parts (products) are not destroyed during the testing process (e.g., testing for resonant frequencies). This work has proved that satisfactory results could be obtained by computer simulation employing the Bond Graph methodology.

The results obtained are very encouraging, and they open further possibilities. It would be necessary to continue working on this approach, and to start developing the entire procedure's models (vibration, shock, drop test), as well as the models that would lead to an efficient application of this methodology to a total approach to the Packaging Distribution Design process. Development of these models would be an important issue as the user (designer) would be able to use them directly when developing the model of a packaging system. These models would make use of this methodology easy for package designers, and, therefore, creating such a library seems to be essential.

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Appendix 1- Characteristics of Lansmont vibration test machine

LANSMONT MODEL 7000-10 VIBRATION TEST MACHINE	
Standard Table Size	48 in. (122 cm) square mounting surface with threaded inserts (3/8 – 16 standard) (M10-1.5 optional), (1/2 – 13 optional)
Standard Table Type	Deeply ribbed aluminium casting with damped top plate
Optional Tables	60 x 98 in. (152 x 249 cm) mounting surface with threaded inserts (3/8 – 16 standard)
Optional table Weight	2175 lb. (987 kg.)
Hydraulic Actuator:	
Pressure rating	3000 psi max. (211 kg/cm2)
Stall Force @ 2500 psi Normal operating pressure	7640 lb. (3465 kg.)
Resisting Moment	57,600 in.-lb. (66,360 cm.-kg.)
Stroke	2.5 in. (6.4 cm.) peak to peak
Piston Weight	105 lb (47.6 kg)
Bearing Type	Hydrofilm composite
Fluid Compatibility	MIL STD 5606 or petroleum-based hydraulic fluid
Mechanical Connections:	
Piston to table	Piston rigidly bolted to table through adapter ring (6 ea. 5/8 – 18 socket head bolts)
Actuator to base	Actuator assembly held to base by eight high strength bolts (5/8 – 18)
Electrical Connections:	
DCDT (Position Feedback)	6 Pin Male (MS3106A-14S-6P)
Differential Pressure	4 Pin Male (PT06AC8-4S)
Servo Valve	4 Pin Male (MS3106A-14S-2P)
Dimensions:	23.5 in. High x 8 in. dia. (59.7 cm x 20.3 cm)
Hydraulic Power Supply	
Pressure rating	3000 psi (211 kg/cm2) max.
Normal operating pressure	2500 psi (176 kg/cm2)
Flow rate	10 gpm (37.9 L/min.) nominal
Filtration	25 micron absolute 10 micron nominal
Recommended Fluids	Shell Tellus 46 Mobil DTE 25 Sun Sunvis 921 Standard Rykon 21 Chevron EP-9
Accumulator pre-charge	2000 psi (141 kg/cm2)
Motor	15 HP (11.2 kw), 1800 RPM (1500 RPM @ 50 Hz)

Utility requirements: Power Cooling water Connections: Mechanical Electrical Power control Dimensions Weight	230/460 VAC 3 Phase (380 VAC optional) 42/21 amp 6 gpm (22.7 L/min) at 10 psi (0.7 kg/cm2) minimum JIC 37° male for hydraulic lines: 2 ea 3553-8-8 (Parker) 1 ea 3553-4-4 (Parker) Garden hose fitting for cooling water 1ea 71GH-12-12 1ea 83GH-12-12 3 phase conduit connector 7 Pin male (MS3102A-16S-1P) 441/2 in. high x 311/2 in. wide x 331/4 in. deep (113 cm x 80 cm x 85 cm) 1200 lb. (550 kg)
Seismic Base Weight Nominal Envelope Dimensions Optional Seismic Base: Weight Dimensions Optional One-G Supports Type Utility requirement	 8000 lb. (3629 kg.) 12.25 x 66.5 x 66.5 in. (31x169x169 cm) 14000 lb. (6351 kg.) 9x60.375x90.25 in. (22.9x153.4x229.2 cm) Allows testing of specimens weighing up to 4000 lb. 81815 kg) for 14000 lb. Seismic Rubber air bladder Plant low pressure air at 80 psi (5.6 kg/cm2) and 0.5 scfm

Appendix 2 - Mechanical properties of Plexiglas - PLASPEC™ Materials Selection Database [12]

MECHANICAL PROPERTIES

TY=	Tensile Strength at Yield	: 10.200E03 PSI
TB=	Tensile Strength at Break Point	: 10.200E03 PSI
EY=	Elongation at Yield	: 4.50 %
EB=	Elongation at Break	: 4.50 %
DL=	Deflection under load	: 0.30 %

MATERIAL DATA [13]:
Acrylic/Plexiglas weight per cubic inch = .04048583 lbs.
F50 breakage test (.25 in): 18 lbs.
Light transmission: 92%

Glass weight per cubic inch = .08439738 lbs.
F50 breakage test (.25 in): 1 lb.
Light Transmission: 70%

ADDITIONAL ACRYLIC/PLEXI GLASS DATA:
Specific Gravity: 1.19
Tensile strength:
rupture= 9,000-11,000 PSI< elongation, rupture % 4.0-4.8
Modulus of elasticity = 400,000 - 500,000 PSI
Flexural Strength:
rupture = 14,000 - 16,500 PSI
modulus of elasticity = 475,000 PSI
Compressive strength:
yield = 18,000 PSI
Modulus of Elasticity = 400,000 - 480,000 PSI
Compressive deformation under load:
4000 PSI, 122 F.,24 hr. = 0.7-0.8%

MATERIAL DATA [14]:
Acrylic/Plexiglas weight per cubic inch = .04048583 lbs.
F50 breakage test (.25 in): 18 lbs.
Light transmission: 92%
Additional Acrylic/Plexi Data:

Specific Gravity 1.19
Tensile strength
rupture= 9,000-11,000 PSI<
elongation, rupture % 4.0-4.8
modulus of elasticity = 400,000 - 500,000 PSI

Flexural Strength
rupture = 14,000 - 16,500 PSI
modulus of elasticity = 475,000 PSI

Compressive strength

yield = 18,000 PSI
Modulus of Elasticity = 400,000 - 480,000 PSI

Compressive deformation
under load
4000 PSI, 122 F.,24 hr. = 0.7-0.8%
Glass weight per cubic inch = .08439738 lbs.
F50 breakage test (.25 in): 1 lb.
Light Transmission: 70%